CONTRUBUTION OF FRAME BAY OF A BUILDING AGAINST LATRAL LOAD

M. Monjur Hossain*[1], M.Kamruzzaman Mahfuz[2] , Imran Uddin Mahmud[3] ,Zobaer Saleheen[4]

[1] *Professor, Dept. Of Civil Engg, Ahsanullah University of Science & Technology, Bangladesh, e-mail: drmonjur@yahoo.com*
[2] *Structural Support Engineer, ARUP, e-mail: mahfuz.ce@gmail.com*
[3] *Inspector of work, SARM Associates Ltd, e-mail: imranmahmud.ce@gmail.com*
[4] *Jr. Structural Engr., SARM Associates Ltd, Bangladesh, e-mail: shauravce100@gmail.com*

ABSTRACT

From a broad viewpoint, the analysis of every structure is approximate, because it is necessary to make certain assumptions in order to carry out the analysis. However, if proper judgment is exercised in making the assumptions upon which the analysis of a given structure is based, the resultant errors will be small. There should be some comparison between the results of analysis done by approximate method and comparatively accurate method (e.g.-computer analysis & shear rigidity). A computer analysis is more accurate, and better suited to analyze complex structures. For structural types that occur commonly one may take advantage of approximate methods of analysis. Such as for lateral load analysis, the commonly used approximate method is portal method. On the other hand, computer analysis is comparatively more accurate. Any weak frame in the structure, will not share the lateral load like other stronger frames of that structure. The objective of this study is to compare the results of frame contribution between the conventional methods and comparatively accurate methods of analysis of a building structure against lateral load for which, a simple structure have analyzed by different methods to examine the relative share of frames against the lateral load like wind & earthquake. The comparison shows wide variation between portal method & shear rigidity method. Finally the STAAD.Pro V8i was used to analyze the same structure which justifies the superiority of shear rigidity method in frame contribution.

Keywords: *Lateral loads, earthquake, shear rigidity method, STAAD Pro, tributary area method*

1. INTRODUCTION

Gravity loads are normally considered as the dead load of the structures including both self-weight of structure, super imposed dead load on structures & service live loads as per design criteria. Lateral loads are normally two types. Wind loads & Earthquake loads. Wind loads are forces on a structure arising from the impact of wind on it. Earthquake or seismic load is the total force that an earthquake exerts on a given structure. It happens at contact surfaces of a structure either with the ground, or with adjacent structures, or with gravity waves from tsunami (Smith & Coull, 1991).

In high rise buildings due to the substantial amount of increase in gravity loading i.e. increased self-weight or live load, the column size of the structure is gradually increased in lower floors. The footing size or depth of foundation also increased with increasing number of floors. Wind load on a tall building acts not only over a large building surface but also with greater intensity at greater height and with a larger moment arm about the base than on a low rise buildings. For buildings up to about 10 stories the design is rarely affected by wind loads (Smith & Coull, 1991). This is more so for lower value of length vs. width ratio. Above this height however the increase in size of some structural members and the possible rearrangement of the structures may have to be done to tackle wind loading. This also incurs a cost premium that increases progressively with height. In earthquake regions, any inertial loads from the shaking of the ground may well exceed the loading due to wind and, therefore be dominant in influencing the building's structural form, design & cost. Although now-a-days with the innovations in architectural treatment, increase in strength of materials and advances in method of analysis permits us to make the tall building lighter and cost efficient.

Approximate method used for analysis for lateral (wind & earthquake) load on a structure is generally the Portal Method. Approximate analysis of hyper static structures provides a simple means of obtaining quick solutions for preliminary designs. It is a very useful process that helps to develop a suitable configuration for final analysis of a structure, compare alternative designs & provide a quick check on the adequacy of structural designs. This method makes use of simplifying assumptions regarding structural behaviour so as to obtain a rapid solution to complex structures. Three dimensional structure is broken down to two dimensional frames on each of which the share of total load is generally assumed on the basis of tributary area. There is no way in portal method to allocate the share of total lateral load on the basis of weak/strong bays (e.g.- weak bays having missing structural elements due to architectural consideration, irregularity). In this method the assumption of tributary area made at the beginning of analysis seems not logical. Selection of member size on the basis of these results may mislead the computer analysis. In this study frame contribution against lateral load by commonly used portal method is compared with computer analysis and shear rigidity method. Computer outputs don't provide frame contribution directly so that the analysed outputs being converted to work done by Strain energy due to bending & Moment factor concept.

2. THEORETICAL CONSIDERATION

There are several approximate methods for the lateral load analysis of building frames (Mahfuz et al, 2015). The most used one of these is the Portal method. It has the advantage of being simple and therefore less time consuming, with less chance of making errors in the calculations. In this paper interest is limited to comparing the result among tributary area method (depending on spacing of columns), shear rigidity method & computer aided analysis in connection with sharing of horizontal load by frames in a building.

2.1 Portal Method

This method is satisfactory for buildings up to 25 stories, hence is the most commonly used approximate method for analysing tall buildings (Schueller, 1977). The following are the simplifying assumptions made in the portal method:

- The point of inflection occurs at the centre of each girder/beam.
- The point of inflection occurs at the centre of each column.
- The total horizontal shear at each storey is distributed between the columns of that storey in such a way that each interior column carries twice the shear carried by each exterior column.

2.2 Shear Rigidity Method

A first step in approximate analysis of a rigid frame is to estimate the allocation of the external horizontal force to each bent. For this it is usual to assume that the floor slab is rigid in plane and therefore, constrain the horizontal displacements of all vertical bents at a floor level to be related by the horizontal translations and rotation in floor slab (Smith & Coull, 1991).

2.2.1 Symmetric Plan Structure Subjected to Symmetric Loading

A symmetric structure subjected to symmetric loading (Fig 1) translates but doesn't twist. From the assumption of slab rigidity, the bents translate identically. The total external shear at a level will be distributed between the bents in proportion to their shear rigidity (GA) at that level. Now it may be obtained for level i in a bent by simply using,

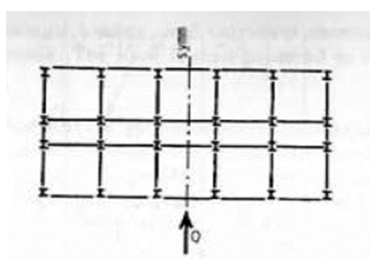

Figure 1: Plan of a symmetric rigid frame

$$GA = \frac{12E}{hi\left(\frac{1}{G}+\frac{1}{C}\right)} \qquad (1)$$

In which,
h_i is the height of storey i,
$G=\Sigma(I_g/L)$, for all the girders of span L across floor i of the bent, where I_g is the moment of inertia of all girders of the bent i,
$C=\Sigma(I_c/L)$, for all the columns of height L in storey i of the bent, where I_c is the moment of inertia of all columns of the bent.
E is the modulus of elasticity.

2.2.2 Asymmetric Plan Structures

The effect of lateral loading on a structure having an asymmetrical plan is to cause a horizontal plane torque in addition to its transverse shear. Therefore the structure will twist as well as translate.
Referring to the asymmetric structure shown in fig 2, and defining the location of centre of shear rigidity of the set of parallel bents in storey i, relative to an arbitrary origin O, as given by

$$x = \left[\frac{\sum (GA) xj}{\sum (GA)} \right] \tag{2}$$

An estimate of the shear Qji carried by bent j at level i is given by

$$Qji = \frac{Qi(GA)ji}{\sum (GA)i} + \frac{Qiei[(GA)c]ji}{\sum [(GA)c^2]i} \tag{3}$$

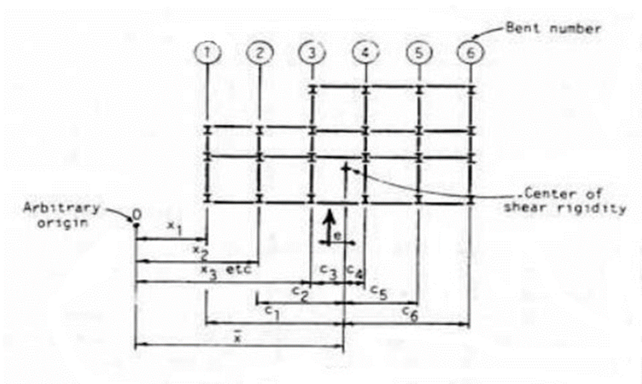

Figure 2: Plan of an Asymmetric Rigid frame

In which for level i
Qi is the total shear of floor I,
(GA)ji is the shear rigidity of bent j in storey i,
ei is the eccentricity of Qi from centre of shear rigidity in storey i,
cj is the distance of bent j from the centre of shear rigidity,
And the two summations refer to the full set of bents parallel to the direction of loading. The signs of c & e are same when they are on the same side of centre of shear rigidity.

2.3 Strain Energy in Bending: (Popov, 1998)

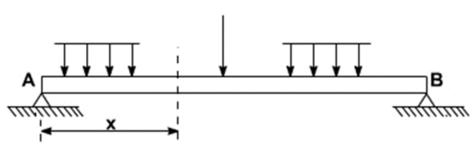

Fig 3: A Simple Beam

The expression of strain energy is derived as,

$$U = \int_0^L \frac{M^2}{2EI} dx \tag{4}$$

Where M is moment due to applied load, E is modulus of elasticity & I is moment of inertia. To calculate strain energy of a frame, taking moment value from analysis of the structures, E from material property, I, L & dx from geometric properties. A sample calculation is shown in analysis methodology.

2.4 Moment Factor λ :

Computer analysis outputs don't provide the individual frame contribution against lateral load, so that a simple scalar factor is used as Moment scalar factor, λ (Mahfuz et al, 2015).
Moment factor λ = (Area of Moment diagrams) / (Stiffness) for a frame.
A sample calculation is shown in analysis methodology.

3. ANALYSIS METHDOLOGY

A simple frame structure as shown below is used for analysis in this study.

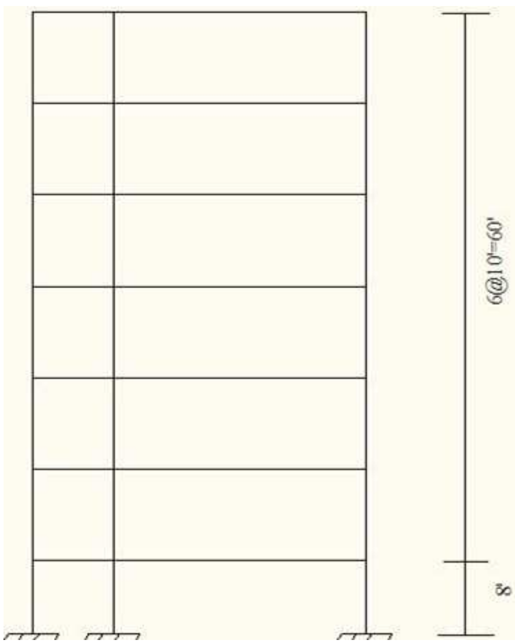

Fig 4: Beam-Column Layout (For 1st to 5th Floor)

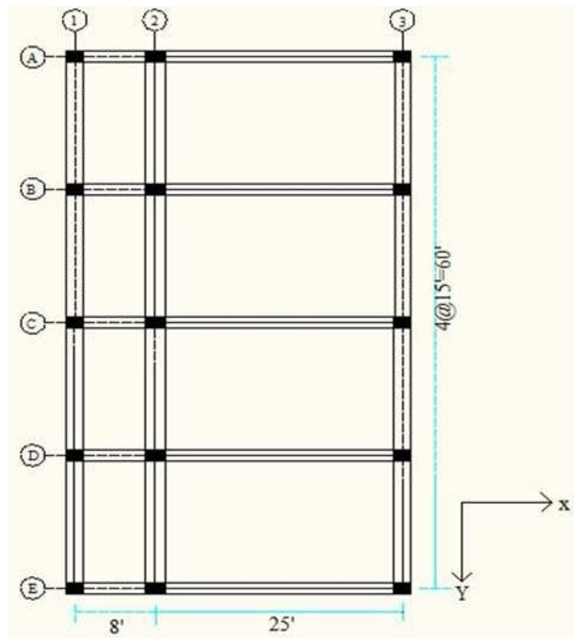

Fig 5: Elevation of frame A

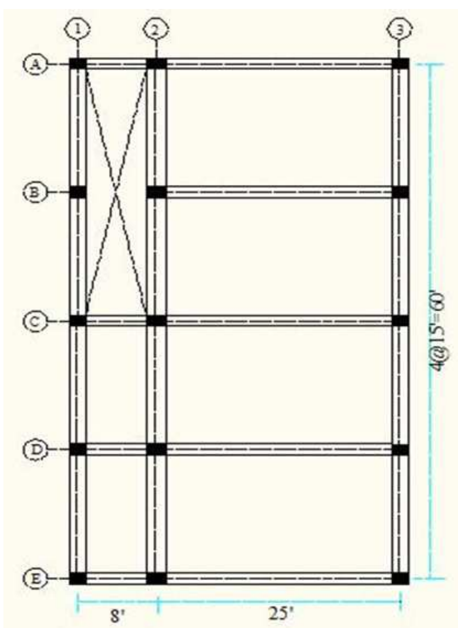

Fig 6: Beam-Column Layout (For GF)

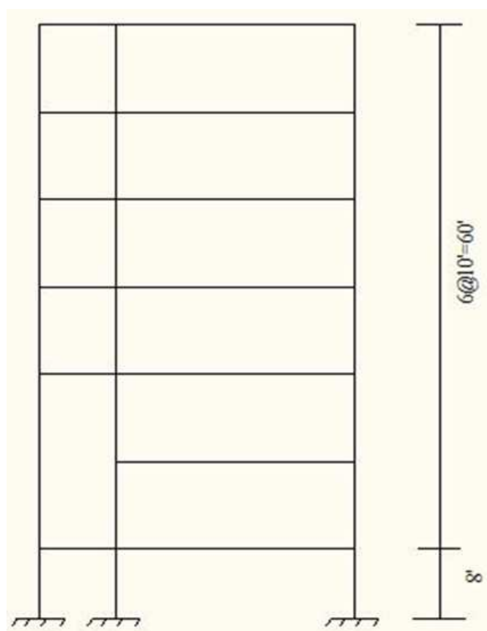

Fig 7: Elevation of frame B

At first, Wind load was calculated according to BNBC then the design components (moment and shear) due to wind load were analyzed by portal method.

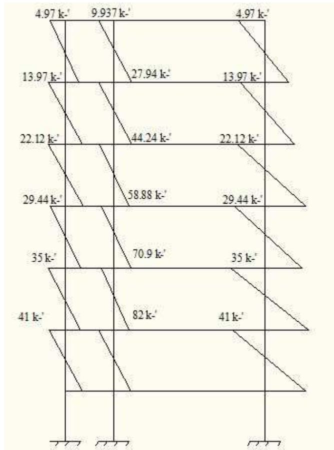

Fig 8: Column BMD due to wind load.

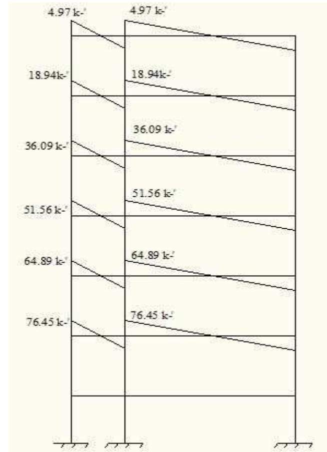

Fig 9: Beam BMD due to wind load.

Then gravitational load was calculated & analyzed by vertical load method .

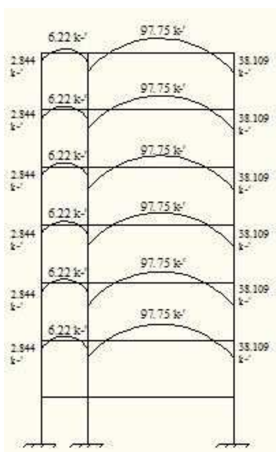

Fig 10: Beam BMD due to gravity load.

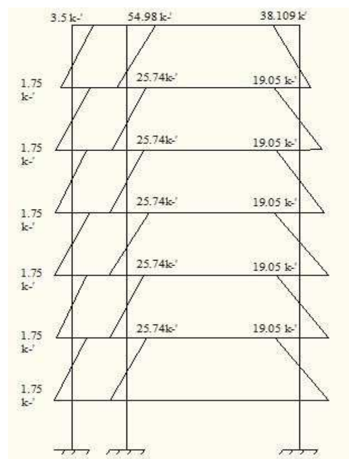

Fig 11: Column BMD due to gravity load.

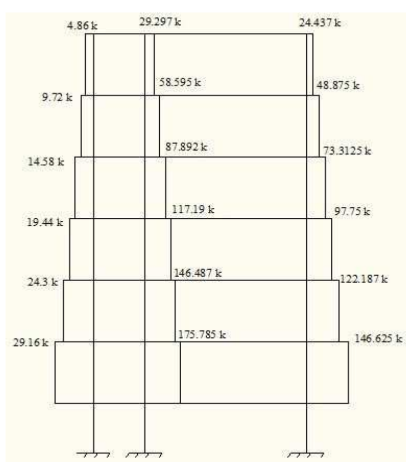

Fig 12: Column AFD due to gravity load.

These were used to determine preliminary size of the structural members.

From the preliminary size earthquake load was calculated. Then the shear rigidity method was applied to determine the contribution of lateral load taken by each bent. Sample calculation is shown below Shear rigidity of Frame A, B, C, D, E (From 1^{st} to 5^{th} Floor).

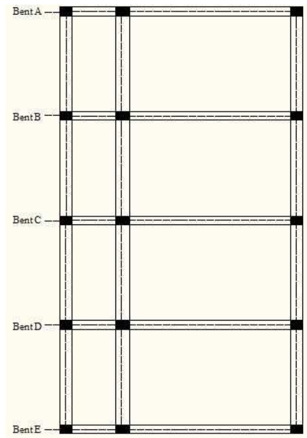

Fig 13: Symmetric plan in X direction (From 1^{st} to 5^{th} Floor)

Contributing Element:
Beam: 10"x 20"

$$G=\Sigma (I_g/L) = \frac{\frac{10*20^3}{12}}{8*12} + \frac{\frac{10*20^3}{12}}{25*12} = 91.67 \text{ in}^3$$

Column: 15" x 20”, 15" x 24"

$$C= \Sigma(I_c/L) = \frac{\frac{15*20^3}{12}*2}{10*12} + \frac{\frac{15*24^3}{12}}{10*12} = 310.67 \text{ in}^3$$

So Shear rigidity, $GA = \frac{12E}{hi\left(\frac{1}{G}+\frac{1}{C}\right)} = 25517 \text{ k/in}^3$

$GA)_A = 25517$ k/in^3
$(GA)_B = 25517$ k/in^3
$(GA)_C = 25517$ k/in^3
$(GA)_D = 25517$ k/in^3
$(GA)_E = 25517$ k/in^3
So Σ (GA) = 127585 k/in^3
Shear rigidity of Frame A, B, C, D, E (For GF)

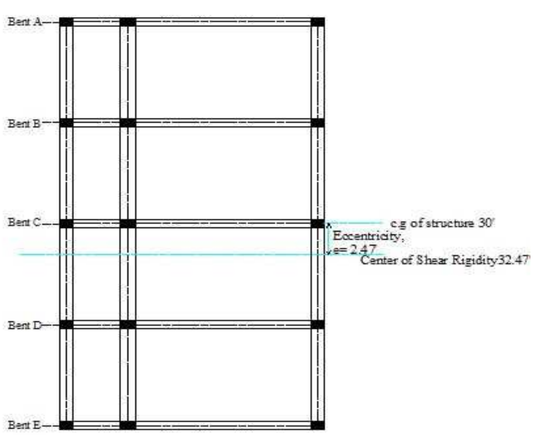

Fig 14: Asymmetric plan in X direction (Ground Floor)

$(GA)_{A\text{-}C\text{-}D\text{-}E}$ = 25517 k/in^3
$(GA)_B$ = 7476.3 k/in^3
So Σ (GA) = 109544.3 k/in^3
From bent A, $\bar{x} = [\frac{\Sigma(GA)xi}{\Sigma(GA)}]$
= {(25517*0) + (7476.3*15) + (25517*30) + (25517*45) + (25517*60)} $*\frac{1}{109544.3}$
= 32.47'
So, eccentricity from center of shear rigidity, e = |30-32.47| = 2.47'

Distribution of EQ Load
For GF (Asymmetric plan)
Bent A

$$Q_{ji} = \frac{9.99*25517}{109544.3} + \frac{9.99*2.47*25517*32.47}{109544.3*\{(32.47-0)^2+(32.47-15)^2+(32.47-30)^2+(32.47-45)^2+(32.47-60)^2\}}$$

= 2.327+0.0811

= 2.408 kip

Bent B

$$Q_{ji} = \frac{9.99*7476.3}{109544.3} + \frac{9.99*2.47*7476.3*(32.47-15)}{109544.3*\{(32.3-0)^2+(32.3-15)^2+(32.3-30)^2+(32.3-45)^2+(32.3-60)^2\}}$$

= 0.694 kip

Bent C

$$Q_{ji} = \frac{9.99*25517}{109544.3} + \frac{9.99*2.47*25517*(32.47-30)}{109544.3*\{(32.3-0)^2+(32.3-15)^2+(32.3-30)^2+(32.3-45)^2+(32.3-60)^2\}}$$

= 2.333 kip

Bent D

$$Q_{ji} = \frac{9.99*25517}{109544.3} - \frac{9.99*2.47*25517*(45-32.47)}{109544.3*\{(32.3-0)^2+(32.3-15)^2+(32.3-30)^2+(32.3-45)^2+(32.3-60)^2\}}$$

= 2.295 kip

Bent E

$$Q_{ji} = \frac{9.99*25517}{109544.3} - \frac{9.99*2.47*25517*(60-32.47)}{109544.3*\{(32.3-0)^2+(32.3-15)^2+(32.3-30)^2+(32.3-45)^2+(32.3-60)^2\}}$$

= 2.257 kip

Similar Procedure was done for wind load also. A computer analysis was done accordingly & the results obtained from different modes of analysis were compared to draw a final conclusion. To used computer analysis, Strain energy due to bending & Moment factor concept is used, first a single frame's total work done is calculated & than it compared with total structure's work done against lateral load by both concept. Sample calculation is shown here only for frame A, Strain energy due to bending.

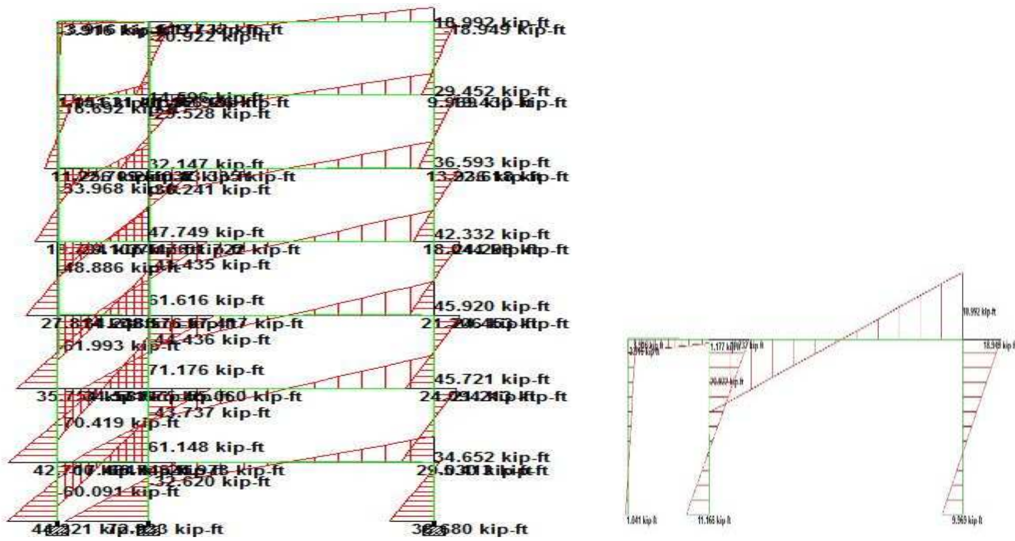

Fig 15: Bending Moment Diagram for Frame A due to EQ in X-direction load case using STAAD.Pro V8i

Level 7 of frame A: calculation for Strain Energy

EQ in X-direction.
Modulus of Elasticity, E is same for all members. So, E can be ignored
Consist two beams, which have same sectional properties but different span length 8' & 25'.

The moment of inertia for 8' & 25' span, I = $\frac{bh^3}{12} = \frac{10x24^3}{12\ x\ 12^4}$ = 0.555 ft^4.

Three column in which end bay column have same cross sectional properties, and interior column have different sectional properties.

I/L for column End bay column or column 1 &3, I = $\frac{bh^3}{12} = \frac{15x20^3}{12\ x\ 12^4}$ = 0.482 ft^4

I/L for column Internal column or column 2, I= $\frac{bh^3}{12} = \frac{15x24^3}{12\ x\ 12^4}$ = 0.833 ft^4

Strain Energy in 8' span beam = $(\int_0^{5.6} \frac{3.92^2}{2EI}dx + \int_{5.6}^{8} \frac{1.18^2}{2EI}dx)$ = 77.524 k-ft.

Strain Energy in 25' span beam =$(\int_0^{13.1} \frac{20.92^2}{2EI}dx + \int_{13.1}^{25} \frac{13.99^2}{2EI}dx)$ = 9031.123 k-ft

Strain Energy in column 1 =$(\int_0^{2.1} \frac{1.04^2}{2EI}dx + \int_{2.1}^{10} \frac{3.92^2}{2EI}dx)$ = 128.284 k-ft;

Strain Energy in column 2 =$(\int_0^{3.61} \frac{11.17^2}{2EI}dx + \int_{3.61}^{10} \frac{19.73^2}{2EI}dx)$ = 1762.89 k-ft.

Strain Energy in Column 3 =$(\int_0^{3.45} \frac{9.97^2}{2EI}dx + \int_{3.45}^{10} \frac{18.95^2}{2EI}dx)$ = 2795.7 k-ft.

Total Strain Energy in Level 7 of Frame A, =77.524 + 9031.123 + 128.284 + 1762.89 +2795.7= 13796 k-ft

Similarly procedure is applied for rest of the level
Finally the Strain Energy in frame A for EQ+X Load case = 520925.2 k-ft.
Similar calculation procedure for other frame in same direction
Strain Energy in Frame B = 573747.2 k-ft.
Frame C =655966.7 k-ft.
Frame D = 590672.3 k-ft.
Frame E =476223 k-ft.

Total Strain Energy due to EQ load in X-Direction = 2817534 k-ft.
Sample calculation shown there only for frame A, Moment factor

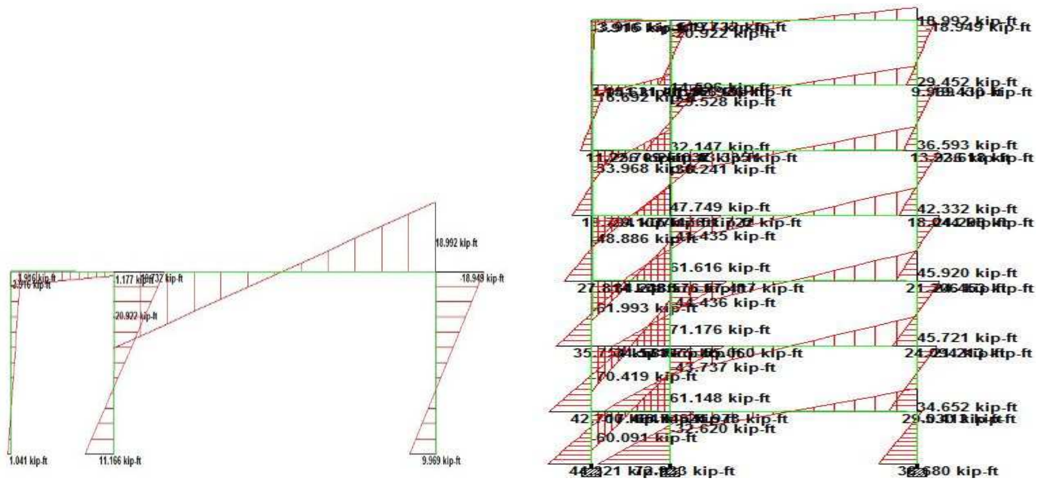

Fig 16: Bending Moment Diagram for Frame A due to EQ+X Load Case Using STAAD.Pro V8i

Level 7 of frame A, calculation for λ
EQ in X-direction,
Consisting of two beams, which have same sectional properties but different span lengths 8' & 25'.

The I/L for 8' span, = I/L = $\frac{bh^3}{12}$ / (L) = $\frac{10x24^3}{12 \times 12^4}$ /8 = 0.0694 ft^3.
For 25'' span= I/L = $\frac{bh^3}{12}$ / (L) = $\frac{10 \times 24^3}{12}$ / 25= 0.0222. ft^3.

Three columns in which, End bay columns have same cross sectional properties, on the otherhand interior columns have different sectional properties.

I/L for column End bay column or column 1 &3, I/L = $\frac{bh^3}{12}$ / (L) = $\frac{15x20^3}{12 \times 12^4}$ /10 = 0.0482 ft^3
I/L for column internal column or column 2, I/L = $\frac{bh^3}{12}$ / (L) = $\frac{15x24^3}{12 \times 12^4}$ /10 = 0.0833 ft^3
Moment factor, λ in 8' span beam = { $\frac{1}{2}$ x (3.92+1.18) x 8} / 0.0694 = 293.95 ft.
Moment factor, λ in 25' span beam= { $\frac{1}{2}$ x 20.92 x 13.1 + $\frac{1}{2}$ x 18.99 x (25-13.1)} / .0222 = 11262'
Moment factor, λ in column 1 = { $\frac{1}{2}$ x 1.04 x 2.1 + $\frac{1}{2}$ x 3.92 x (10-2.1)}/0.0482= 343.9 ft.
Moment factor, λ in column 2= { $\frac{1}{2}$ x 11.17 x 3.61 + $\frac{1}{2}$ x 19.73 x (10-3.61)}/ 0.0833 = 998.79 ft.
Moment factor, λ in Column 3 = $\frac{1}{2}$ x 9.97 x 3.45 + $\frac{1}{2}$ x 18.95 x (10-3.45) = 1644.37 ft.
Total Moment factor, λ in Level 7 of Frame A, = 293.95 + 11626 + 343.9 + 998.79 +1644.37
= 14530.14 ft.

Similarly procedure is applied for rest of the level.
Finally the Moment factor, λ in frame A for EQ+X Load case = 208364 ft.
Similar calculation procedure for other frame in same direction
Moment factor, λ in Frame B = 232251.7 ft.
Frame C = 234183.7 ft.
Frame D = 223522.3 ft.
Frame E = 200540.9 ft.
Total EQ+X Work in X-Direction = 1098863

4. OUTCOMES OF THE RESULTS & COMPARISON

Individual frame contributing against lateral load by different method analysis is shown and compared with each other.

Table 1: EQ load & wind load in X-direction contribution by individual frame by tributary area

	Frame A	Frame B	Frame C	Frame D	Frame E
Total EQ=143.19 kip	17.898	35.795	35.795	35.795	17.898
Percent (%)	12.5%	25%	25%	25%	12.5%
Total WL=130.2 kip	16.28	32.545	32.545	32.545	16.28
Percent (%)	12.5%	25%	25%	25%	12.5%

Table 2: EQ load in X-Direction & wind load in X-Direction contribution by individual frame by Shear Rigidity

Frame	A	B	C	D	E
Total EQ=143.32kip	29.019	27.412	28.949	28.973	28.879
Percent (%)	20.27%	19.138%	20.22%	20.195%	20.17%
Total WL=130.2kip	26.681	24.002	26.563	26.505	26.447
Percent (%)	20.5%	18.40%	20.409%	20.36%	20.315%

Table 3: Moment factor, λ by individual frame for EQ load & wind load in X-direction

Frame	A	B	C	D	E
Total Moment factor(EQ), λ=829049. ft.	158432.3	170185.8	177648.4	170188.8	152594.2
Percentage (%)	19.11 %	20.5 %	21.43 %	20.55 %	18.41 %
Total Moment factor(wind), λ =1497443 ft.	279680.4	307447.2	331798.6	311756.1	266760.2
Percentage (%)	18.67 %	20.53 %	22.15 %	20.84 %	17.80 %

Table 4: Stain energy contribution for individual frame for EQ & wind load in X-direction

Frame	A	B	C	D	E
Total Strain energy (EQ) U = 3585820.8 k-ft	662223.7	735085.4	828991.2	752952.0	606568.4
Percentage (%)	18.46 %	20.5 %	23.11 %	21 %	16.92 %
Total Strain energy (wind) U = 3161992.8 k-ft.	552333.9	629254.9	796995.1	691514.9	491893.8
Percentage (%)	17.46 %	19.90 %	25.20 %	21.88 %	15.56 %

Table 5: Comparison of lateral load contribution in each frame for WL in X-direction by different methods

Method	A	B	C	D	E
Tributary Area	12.5%	25%	25%	25%	12.5%
Shear Rigidity	20.5%	18.40%	20.409%	20.36%	20.315%
Moment factor, λ	18.67 %	20.53 %	22.15 %	20.84 %	17.80 %
Strain Energy, U	17.46 %	19.90 %	25.20 %	21.88 %	15.56 %

Table 6: Comparison of Lateral load contribution in each frame for EQ in X-direction

Method	A	B	C	D	E
Tributary Area	12.5%	25%	25%	25%	12.5%
Shear Rigidity	20.27%	19.138%	20.22%	20.195%	20.17%
Moment factor,λ	19.11 %	20.5 %	21.43 %	20.55 %	18.41 %
Strain Energy, U	18.46 %	20.5 %	23.11 %	21 %	16.92 %

5. CONCLUSION

- In the Portal Frame method (tributary area method) the results depend only on the distance of the frames and height of story. Therefore, this method ignores structural sectional size or stiffness in calculating the share of frame against lateral load. This seems to be a big discrepancy leading to a final design.
- If a frame is weak Portal Frame method (the tributary area method) shows no difference in contributing towards lateral loads.
- In the shear rigidity method the contribution of lateral loads taken by each frame considers the effect of tributary and also on sectional properties of the frame member.
- In the shear rigidity method the contribution of lateral loads taken by each frame considers the effect of inherent weakness of the individual frame in the group if any.
- In the computer aided analyses (strain energy and moment factor prescribed herein), shows relative authentication for shear rigidity method much better than Portal method.
- Finally it can be concluded that results obtained by shear rigidity method is next to computer analysis result and can be used for design purpose for buildings having non identical frames.
- For structure which is complex, computer aided solution is recommended.

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