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NUMERICAL STUDY ON UPLIFT CAPACITY OF STRIP PLATE ANCHOR IN SPATIALLY RANDOM COHESIVE SOIL

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ABSTRACT

Plate anchors are commonly used to resist uplift forces of both offshore and on-land structures. Numerous studies have been carried out by several researchers to date to evaluate the ultimate uplift capacity of plate anchor placed in cohesive soil, not considering the effects of spatial variability of soil strength. The soil characteristics may vary significantly even within a seemingly homogeneous soil layer, which could broadly impact the capacity of plate anchors. Therefore, in this study, finite element limit analysis is carried out to investigate the impact of spatial variability of cohesive soil on the uplift capacity of horizontal strip anchors. The results of the current finite element model are validated with the existing experimental and numerical outcomes prior to detail parametric studies. In the current study, the spatial variation of undrained shear strength was incorporated by considering several combinations of coefficient of variation (COV_{Cu}), and dimensionless isotropic correlation length ratio (Θ_{lnCu}). Based on the random field theory of Karhunen-Loeve expansion, Monte Carlo Simulation with 1000 repetitions are conducted for each set of COV_{Cu} and Θ_{lnCu}. The uplift capacity of the strip anchor is expressed as a dimensionless mean break-out factor. The effects of COV_{Cu} and Θ_{lnCu} on the mean break-out factor are estimated. The numerical results reveal that both COV_{Cu} and Θ_{lnCu} significantly affect the uplift capacity of horizontal strip anchors. From the numerical analyses, it is found that the horizontal strip anchor requires higher safety factor compared to the conventional factor of safety in case of higher COV_{Cu} and Θ_{lnCu} values; whereas, the conventional factor of safety could be used for smaller values of COV_{Cu} and Θ_{lnCu} . The outcomes of this study confirm that the spatial variability of soil strength should be considered for the safe design of the plate anchor.

Keywords: *Break-out factor, Spatial variation of undrained shear strength, Numerical limit analysis, Monte Carlo Simulation, Probabilistic failure analysis*

1. INTRODUCTION

The design of the foundation supporting transmission towers, mooring systems, offshore structures, and storage tanks requires resistance against uplifting force due to the application of outwardly directed loads (e.g., wind loads, earthquake loads, seepage forces) above or below the ground level. Anchors are used to resist these uplift forces. The uplift capacity of the anchor is dependent on the dimensionless break-out factor. Several studies have been conducted over the past few decades to estimate the ultimate uplift capacity of anchors encased in clay either by laboratory tests or analytical solutions.

Several researchers published empirical relationships of the critical break-out factors in horizontal anchors based on laboratory test results (Meyerhof & Adams, 1968; Vesic, 1971; Meyerhof, 1973; Das, 1978; Das, 1980). Meyerhof & Adams (1968) found the critical break-out factor of value 10 for the circular anchor in the case of soft clay, which was not corrected for suction. Vesic (1971) used the cavity expansion formula and found the critical break-out factor in between 9-10 in soft clay. Meyerhof (1973) proposed the critical break-out factor 8 in the case of horizontal strip anchors embedded in clay. From the small-scale laboratory tests, it was found that the break-out factor increased with the increase of embedment ratio linearly up to a value of 6 (Das, 1978). In another study, he proposed an empirical equation for determining the break-out factor and suggested a procedure to estimate the uplift capacity of horizontal anchor (Das, 1980).

Several numerical studies were also conducted to investigate the behaviour and stability of strip anchors embedded in isotropic as well as both homogeneous and non-homogeneous cohesive soil (Rowe & Davis, 1982; Merifield et al., 2001; Martin & Randolph, 2001; Yu et al., 2011). The numerical analysis performed in the case of non-homogeneous cohesive soil, assumed that the strength of soil increased linearly with depth. However, to the authors' knowledge, the impacts of soil heterogeneity on the performance and capacity of horizontal strip anchors are not investigated thoroughly to date. The spatial variability of the soil parameters (i.e., undrained shear strength) plays a vital role in the stability of the anchor and therefore, a substantial investigation is deemed essential. The aim of this study is thus to investigate the effects of heterogeneity of soil strength in spatially random cohesive soil on the uplift capacity of horizontal strip anchor plate through finite element limit analyses.

In the current study, the uplift capacity of horizontal strip anchors embedded in homogeneous purely cohesive soil are estimated for several embedment ratios (H/B, the ratio between embedment depth to plate width) through the OPTUM G2 finite element model. The obtained uplift capacity is converted to the dimensionless parameter by using break-out factor F_c, and defined as

$$F_c = \frac{1}{C_u}\left(\frac{Q_o}{B \times L}\right) \tag{1}$$

Where Q_o = net ultimate uplift capacity, B = width of the anchor, L = length of the anchor (i.e., L = 1 m for 2D analyses), C_u = undrained shear strength of the cohesive soil.
The obtained numerical results are compared with the existing experimental and numerical outcomes to validate the developed OPTUM G2 model.

Further, the uplift capacity of the anchor is analyzed using probabilistic numerical limit analyses to incorporate the stochastic spatial variation of undrained shear strength. The Monte Carlo Simulation is conducted based on the random field theory of the Karhunen-Loeve expansion. Two parameters (COV_{Cu} and Θ_{lnCu}) are used to incorporate the spatial variability of undrained shear strength of cohesive soil.

The coefficient for the variation of undrained shear strength COV_{Cu}, is defined as

$$COV_{C_u} = \frac{\sigma_{C_u}}{\mu_{C_u}} \tag{2}$$

Where: σ_{Cu} = standard deviation of undrained shear strength, μ_{Cu} = mean of undrained shear strength
The spatial correlation length is the distance over which the spatial undrained shear strength will be correlated. At any point, the undrained shear strength will associate with the adjacent point in both vertical and horizontal directions; the magnitude of the correlation relies on the distance between the two points. The isotropic correlation length (θ_{lnCu}) is converted dimensionless based on the study of Griffiths et al. (2002) and Kasama & Whittle (2011), defined as

$$\Theta_{lnCu} = \theta_{lnCu}/B \tag{3}$$

Where: Θ = dimensionless correlation length ratio

Phoon & Kulhawy (1999) suggested the horizontal and vertical correlation lengths (scale of fluctuations) of undrained shear strength of 40-60 m and 0.8-6 m, respectively. The typical coefficient of variation of undrained shear strength will vary in the range of 0.1-0.8 in clay soil (Phoon & Kulhawy, 1999; Duncan, 2000). In this study, isotropic correlation length is considered (horizontal and vertical correlation length are assumed to be equal) with a constant mean undrained shear strength (μ_{C_u}= 22.07 kN/m^2). For the probabilistic analyses, the isotropic correlation length and standard deviation are varied in a single clay layer. For each set of assumed COV_{Cu} and Θ_{lnCu}, Monte Carlo Simulation with 1000 repetitions has been conducted. The standard error of the break-out factor for 1000 repetitions is about $\sqrt{1/1000} = 0.03162 = 3.2\%$ of the standard deviation of each result. For the variation in soil properties, the required design factor of safety is suggested based on the statistical probability analyses.

2. PROBLEM DEFINITION

A horizontal strip anchor of a width, B = 2m, is placed into the homogeneous cohesive soil layer with a constant mean undrained shear strength value of 22.07 kN/m^2. In shallow depth, the uplift capacity increases due to the suction pressure developed between the surface of the anchor and soil (i.e., no breakaway). This pressure diminishes as the embedment ratio increases (Das et al., 1994). Determination of the magnitude of the suction pressure is uncertain as it depends on several factors (e.g., soil permeability, undrained shear strength, and rate of loading). Due to this uncertainty, this study is performed for immediate breakaway conditions (i.e., the soil and anchor interface cannot sustain tension).

3. FINITE ELEMENT MODEL

The finite element limit analysis is carried out using OPTUM G2 to estimate the break-out factor in each embedment ratio varying from 0.5 to 10 (H/B = 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10). The soil is assumed isotropic with an associated Tresca yield criterion. The steel plate is considered rough and weightless. For adopting immediate breakaway conditions, tension cut-off is used at the interface of the rigid plate, where tensile strength, $k_t = 0$, and inclination of the tension cut-off cone, $\varnothing_t = 90°$.

A typical 2-D mesh of horizontal strip anchors with a width of B = 2m, and an embedment ratio of H/B = 9 is shown in Figure 1. The soil boundary is extended 16B in both horizontal and vertical directions. The optimum mesh element is selected (i.e., number of elements = 6000) to ensure the accuracy of the results and reduce computation time.

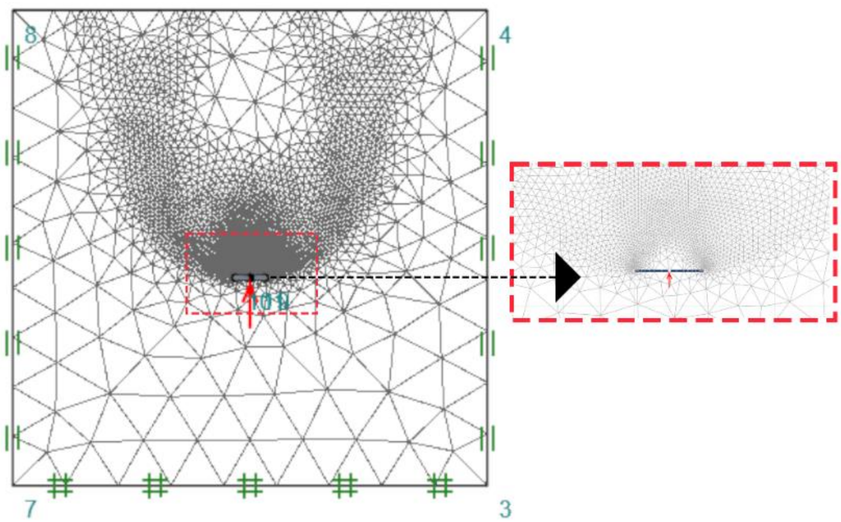
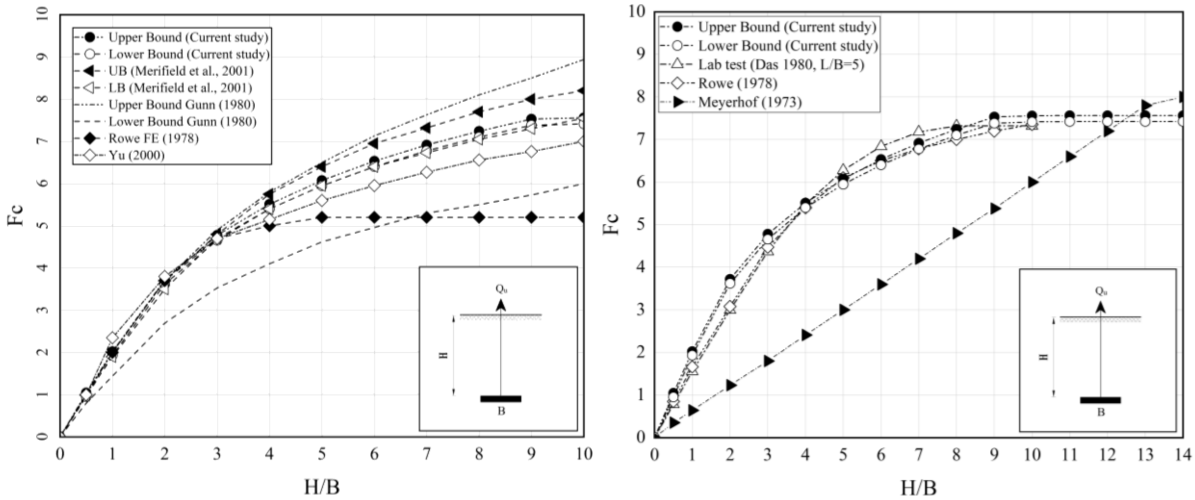


Figure 1: Typical finite element mesh in homogeneous cohesive soil (H/B = 9, B = 2m).

4. VALIDATION OF THE MODEL

Figure 2 compares the present numerical limit analysis results with both existing numerical and experimental outcomes. In Figure 2(a), both upper bound (UB) and lower bound (LB) solutions are in between the numerical results of Gunn (1980) and Merifield et al. (2001). The benefit of using UB and LB limit analyses is that the exact solution is always bracketed between those limits. The current study shows that the exact value of break-out factor is somewhere within ±1% of the UB and LB solutions. The numerical results of Rowe (1978) and Yu (2000) show a deviation from the results of the current study beyond the embedment ratio(H/B) value of 3.

The laboratory results presented in Figure 2(b) are based on small-scale model tests (e.g., the width of the anchor plate is less than 50mm), where the effect of overburden pressure was neglected. From Figure 2(b), it can be seen that the break-out factor obtained from the present study matches quite well with experimental results when H/B > 4. The current study uses strip anchor, while Das (1980) performed the test on rectangular anchors (i.e., L/B = 5), and Rowe (1978) used a rectangular anchor, L/B > 5. Meyerhof (1973) found the value of ultimate break-out factor 8 for strip anchor. Meyerhof's (1973) results are different from the results of the current study as well as other research because the uplift capacity near the footing was reduced due to tensile failure of the clay.



(a) (b)

Figure 2: Break-out factors for horizontal strip anchor in homogenous cohesive clay: (a) comparison with existing numerical analysis; (b) comparison with available laboratory results.

5. NUMERICAL LIMIT ANALYSIS

The upper and lower bound limit analyses are performed using a constant mean undrained shear strength, μ_{C_u} = 22.07 kN/m^2, while the combination of COV_{Cu}, and Θ_{lnCu} are varied using the following ranges:

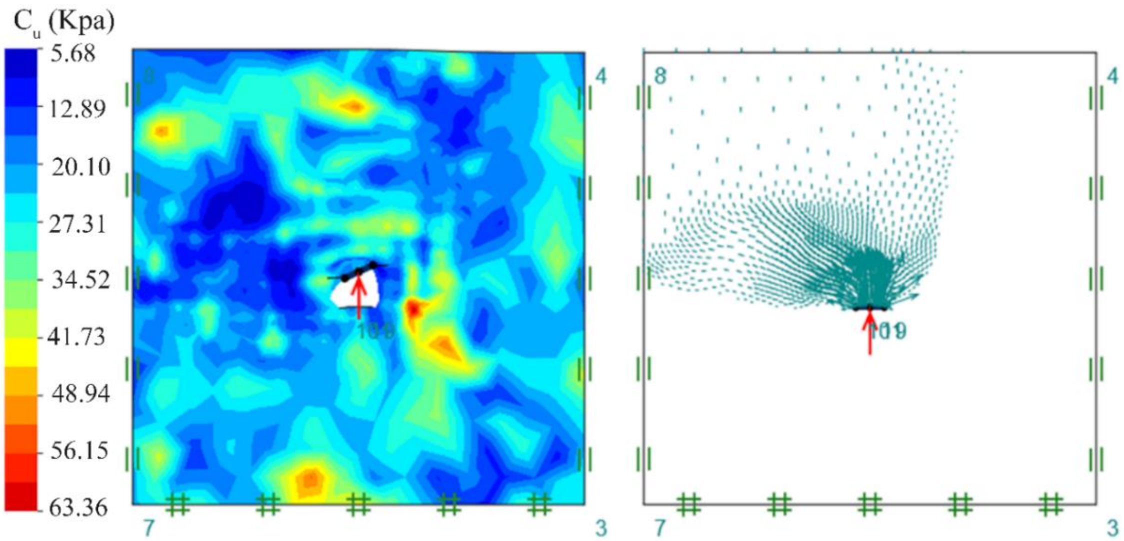
COV_{Cu} = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0
Θ_{lnCu} = 0.125, 0.25, 0.5, 1, 2, 4, 8

The use of lower and upper bound limit analysis ensures the exact solution in between UB and LB results. 1000 Monte Carlo Simulations are analyzed for each combination of coefficient of variation and correlation length. The mean break-out factor is calculated by,

$$\mu_{F_c} = \frac{1}{1000*\mu_{C_u}} \sum_{i}^{1000} (\frac{Q_{fi}}{B}) \qquad i = 1, 2, 3, 4, \ldots\ldots\ldots\ldots, 1000 \tag{4}$$

$$\sigma_{F_c} = \sqrt{\frac{1}{1000-1} \sum_{i=1}^{1000} (F_{Ci} - \mu_{F_c})^2} \qquad i = 1, 2, 3, 4, \ldots\ldots\ldots\ldots, 1000 \tag{5}$$

Figure 3 represents the typical deformed shape at failure corresponding to COV_{Cu} = 0.4, Θ_{lnCu} = 1, and μ_{C_u}= 22.07 kN/m^2, where the correlation length is 2m (horizontal and vertical). The reddish zone indicates firmer soil, whereas the bluish region shows weaker soil. Figure 3(b) shows that the displacement vector follows a non-symmetric failure pattern due to heterogeneity. Equation (4) gives a mean break-out factor, μ_{F_c} = 6.58 (COV_{Cu} = 0.4, Θ_{lnCu} = 1). Due to the heterogeneity (i.e., variation of undrained shear strength, correlation length), the obtained result shows a 12.5% discrepancy from the homogeneous soil in the same embedment ratio, H/B = 9.



(a) (b)

Figure 3: Typical upper bound limit analysis results: (a) deformed shape at failure with COV_{Cu} = 0.4, Θ_{lnCu} = 1; (b) displacement vectors showing non-symmetric behaviour at failure.

5.1 Brief description of Log-Normal Distribution

The log-normal distribution is used to model the variability of undrained shear strength, where ln_{Cu} is normally distributed. The log-normal distribution of σ_{C_u}, μ_{C_u} are given by

$$\sigma_{ln_{Cu}} = \sqrt{\{ln[1 + (COV_{Cu})^2]\}} \tag{6}$$

$$\mu_{ln_{Cu}} = ln\mu_{Cu} - \frac{1}{2}\sigma^2_{ln_{Cu}} \tag{7}$$

The probability distribution function (PDF) is given by

$$f\ (x;\ \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} exp[\frac{-(ln\, x - \mu)^2}{2\sigma^2}], x > 0 \tag{8}$$

Instead of using a normal distribution, the log-normal function is used because it is always non-negative; no meaningless negative value generates in the probability distribution.

6. RESULTS OF LIMIT ANALYSIS

For each combination of input values (i.e., COV_{Cu} and Θ_{lnCu}), the mean and standard deviation of the break-out factors are determined using equations (4 and 5). A 22 bins histogram of 1000 break-out factors is plotted with the 'best-fit' log-normal distribution, shown in figure 4 for the set where COV_{Cu} = 0.4, Θ_{lnCu} = 1.

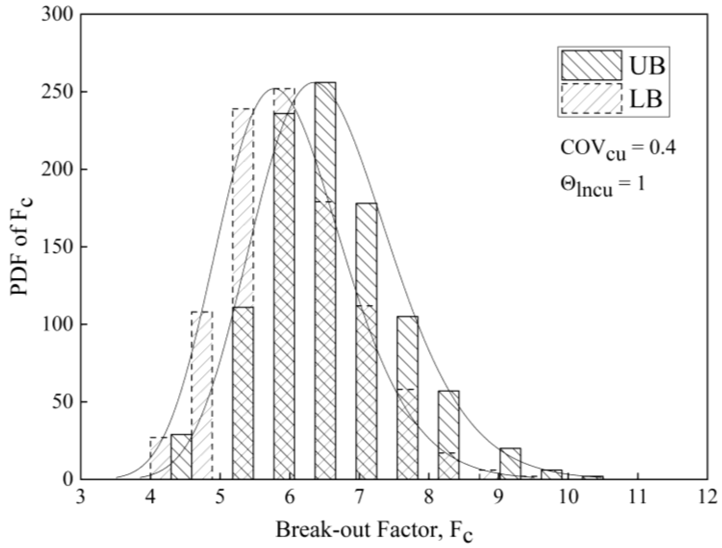
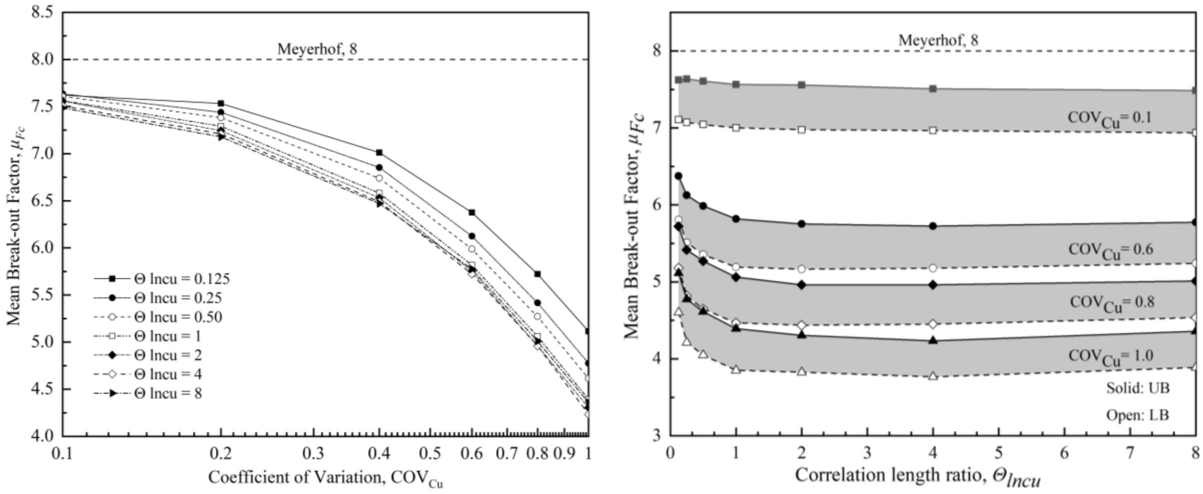


Figure 4: Histogram and log-normal fit for the break-out factors (COV_{Cu} = 0.4, Θ_{lnCu} = 1).



(a) (b)

Figure 5: (a) Mean break-out factor, μ_{Fc} (UB) as a function of spatially variable parameters of undrained shear strength, Θ_{lnCu}, and COV_{Cu}. (b) variation of correlation length ratio with μ_{Fc} (UB, LB).

The variation of mean break-out factor, μ_{Fc} (UB) with Θ_{lnCu} and COV_{Cu}, is shown in figure 5(a). As COV_{Cu} increases, the mean break-out factor decreases. When the COV_{Cu} is small, the mean break-out factors approach the critical break-out factor, $F_c^* = 8$ (design factor). Higher values of COV_{Cu} show a steep fall of μ_{Fc}. The estimated μ_{Fc} decreases with an increase of Θ_{lnCu}. However, this inverse relationship is valid for lower values of Θ_{lnCu}. For higher values of $\Theta_{lnCu,}$ the mean break-out factors are almost identical, and for an increase of Θ_{lnCu}, the mean break-out factor increases slightly. This relation of Θ_{lnCu}, and μ_{Fc} are shown in Figure 5(b). The mean break-out factor decreases slightly for a smaller correlation length and increases for a higher correlation length (i.e., $\Theta_{lnCu} > 2$). For vanishingly small Θ_{lnCu}, the reason for higher μ_{Fc} is because, the weakest path of the soil starts to look for shorter routes and fails through the high strength soil, while the ideal failure path follows the nature of uniform soil.

6.1 Probability of Failure

Failure is assumed to occur when the break-out factor is less than a suggested critical break-out factor, $F_c^* = 8$. For lognormally distributed Fc, the probability of the failure P [Fc < 8/FS] is given by,

$$P\left[Fc \ < \frac{8}{FS}\right] = \ \Phi\left(\frac{ln(8/FS) - \mu_{ln_{Fc}}}{\sigma_{ln_{Fc}}}\right) \tag{9}$$

Where $\Phi\ (\ldots) =$ cumulative normal function.
For a particular set shown in Figure 4, $\mu_{F_c} = 6.581\ (UB)$ and $\sigma_{Fc} = 1.0234\ (UB)$. Equation (6) and (7) gives $\sigma_{ln_{Fc}} = 0.15458$ and $\mu_{ln_{Fc}} = 1.872$ respectively. From equation (9), the probability of failure is found as P [Fc < 8] = 0.9099, indicating a 90.99% probability that the computed uplift capacity will be less than the design capacity when FS =1. The lower bound solution indicates a 97.53% probability that the uplift capacity is smaller than the nominal factor. The exact failure probability will be in between 91 to 97 percent for FS=1.

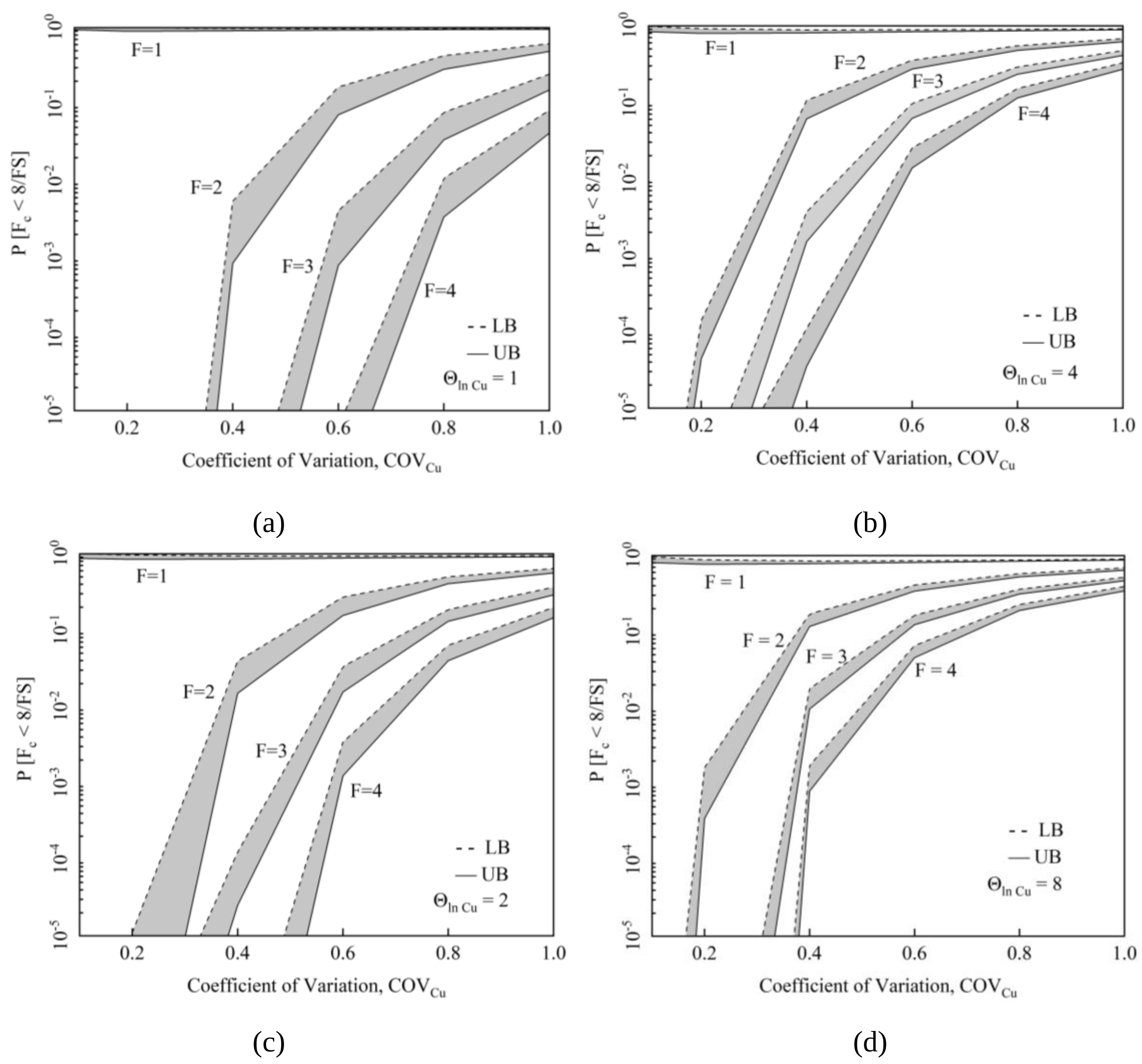
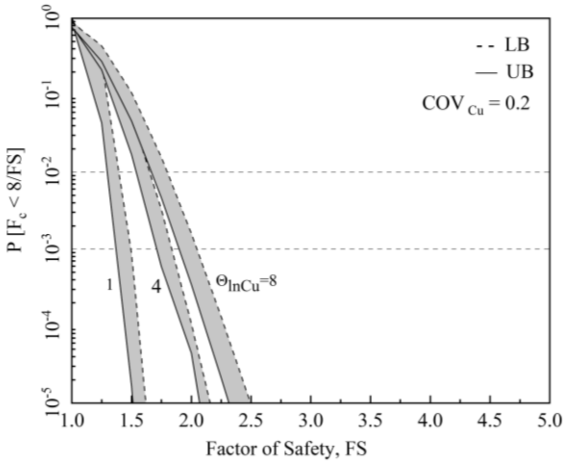


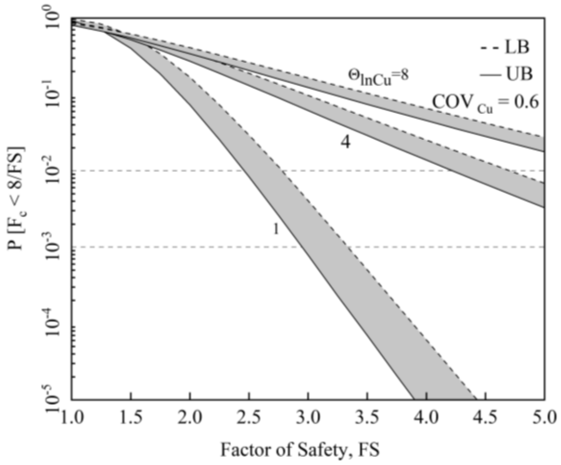
Figure 6: Probability that the mean break-out factor is less than the nominal design factor: (a) $\Theta_{\ln Cu}$ =1; (b) $\Theta_{\ln Cu}$ =2; (c) $\Theta_{\ln Cu}$ =4; (d) $\Theta_{\ln Cu}$ =8.

Figure 6 summarizes how the factor of safety (i.e., FS = 1, 2, 3, 4) affects the probability of nominal design failure as a function of coefficient of variation for several dimensionless correlation length ratio, $\Theta_{\ln Cu} = 1$, $\Theta_{\ln Cu} = 2$, $\Theta_{\ln Cu} = 4$, $\Theta_{\ln Cu} = 8$. The results show that higher FS is required to reduce the failure probability. For cohesive soil with $COV_{Cu} = 0.5$ and $\Theta_{\ln Cu} = 1$, a safety factor of at least three is required, while greater FS is required for large COV_{Cu}. Fig. 6 also shows, in the same $COV_{Cu,}$ larger FS is necessary for an increase of $\Theta_{\ln Cu}$. When $COV_{Cu} = 0.5$ and $\Theta_{\ln Cu} = 8$, required design factor of safety is, FS = 4 with a failure probability in the acceptable level (i.e., P[Fc < 8/FS] < 10^{-2}). For an increase in $\Theta_{\ln Cu}$, the gap between the various FS reduces.

A more detailed stochastic limit analysis with a probabilistic interpretation of design failure is shown in Fig. 7. The permissible failure probability for shallow foundations is advised to be in the range, $P_f = 10^{-2} – 10^{-3}$ (Baecher & Christian, 2005; Bathurst et al., 2011; Phoon et al., 2000). Figure 7 show that P[Fc < 8/FS] is at the acceptable level for $COV_{Cu} < 0.5$, $\Theta_{\ln Cu} < 8$ with a safety factor of 3. When 0.5 < COV_{Cu} <0.8, $0.5 \leq \Theta_{\ln Cu} \leq 8$, a minimum FS of 4 is required. However, in rare cases with $COV_{Cu} \geq 0.8$ and $1 < \Theta_{\ln Cu} \leq 8$, the projected failure probability fails to reach the acceptable limit, $P_f = 10^{-2}$.



(a)



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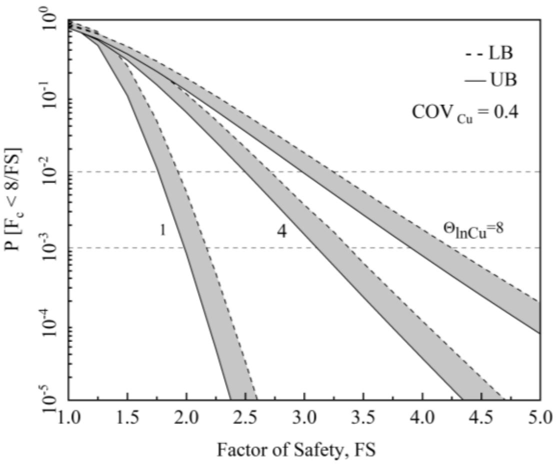
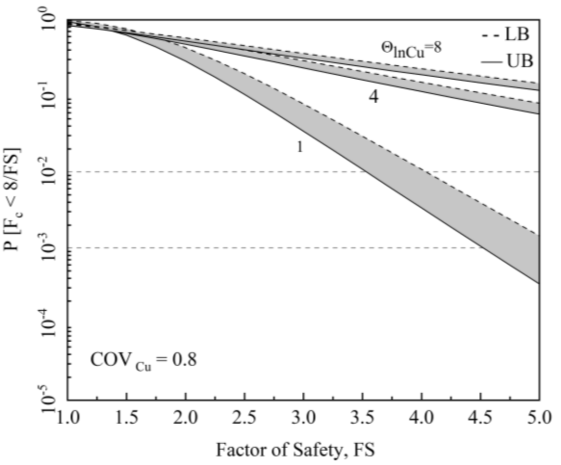


(b)



(d)

Figure 7: Variation of P[Fc < 8/FS] with safety factor for a cohesive soil: (a) $COV_{Cu} = 0.2$, $1 \leq \Theta_{\ln Cu} \leq 8$; (b) $COV_{Cu} = 0.4$, $1 \leq \Theta_{\ln Cu} \leq 8$; (c) $COV_{Cu} = 0.6$, $1 \leq \Theta_{\ln Cu} \leq 8$; $COV_{Cu} = 0.8$, $1 \leq \Theta_{\ln Cu} \leq 8$.

7. CONCLUSIONS

In this study, the numerical limit analysis is carried out to examine the effects of spatial variability of undrained shear strength on the uplift capacity of horizontal strip anchor embedded in spatially random cohesive soil. The coefficient of variation (COV_{Cu}) and dimensionless isotropic correlation length ratio (Θ_{lnCu}) are assumed to represent the spatial variation of undrained shear strength. Based on the outcomes of this study, the following conclusions can be drawn:

(a) The calculated mean break-out factor drops as the coefficient of variation of undrained shear strength increases. An increase in the mean break-out factor was noticed for a vanishingly small dimensionless correlation length ratio. For higher $\Theta_{lnCu,}$ the changes of mean break-out factors are almost identical.

(b) As the coefficient of variation increases, the necessity of higher safety factor increases. A more significant factor of safety (FS) is required when $COV_{Cu} > 0.5$. For the same COV_{Cu}, the gap between the necessary safety factors (i.e., FS > 1) decreases with the dimensionless correlation length ratio.

(c) By investigating the effect of the probability of failure with the safety factor, a factor of safety of three is suggested when $COV_{Cu} < 0.5$and $\Theta_{lnCu} < 8$. A minimum safety factor of four is necessary for a cohesive soil with $0.5 < COV_{Cu} <0.8$ and $0.5 \leq \Theta ln_{Cu} \leq 8$. However, in exceptional cases where $COV_{Cu} \geq 0.8$ and $1 < \Theta_{lnCu} \leq 8$, a higher factor of safety is required.

(d) The impact of correlation length on the probabilistic uplift capacity interpretations is significant, particularly for a cohesive soil with a greater coefficient of variation. More research is needed to examine the effects of anisotropic correlation length and the coefficient of variation of undrained shear strength on the uplift capacity of horizontal plate anchors.

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