NUMERICAL LIMIT ANALYSES ON UPLIFT CAPACITY OF HORIZONTAL STRIP ANCHOR IN SPATIALLY RANDOM FRICTIONAL SOIL

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ABSTRACT

Horizontal plate anchors are essential to resist uplift forces of offshore structures, mooring systems, and transmission towers. Numerous studies have been conducted over the years to determine the ultimate uplift capacity of an anchor embedded in frictional soil, but the effects of spatial variability of soil strength have been overlooked that may have a significant impact on the capacity of the anchor plate. Therefore, this study aims to analyze the effect of spatial variability of frictional soil on the uplift capacity of horizontal strip anchors through numerical limit analysis. In order to validate the developed finite element model, the model results were initially compared with with the available experimental and numerical outcomes. Through parametric studies, the spatial variation of friction angle is taken into account by considering several combinations of coefficient of variation (COV_{φ}) and dimensionless correlation length ratio (Θ_{φ}) of friction angle. Based on the random field theory of Karhunen-Loeve expansion, Monte Carlo Simulation with 1000 repetitions is conducted for a selected range of COV_{φ}, and Θ_{φ} . In shallow depth, the effect of COV_{φ} and Θ_{φ} on the computed mean break-out factors are estimated. The probabilistic numerical analyses reveal that both COVφ and Θ_{φ} significantly influence the uplift capacity of the horizontal strip anchor plate buried in frictional soil. For higher COV_{φ} and Θ_{φ}, the geotechnical design of shallow horizontal anchor requires higher safety factors compare to the conventional factor of safety. In contrast, the conventional safety factor can be used for smaller COV_{φ}. The results of this study verify that, in the geotechnical design of horizontal anchor plates, the effect of spatial variation of friction angle should be considered to ensure safety.

Keywords: *Break-out factor, Spatial variation of friction angle, Numerical limit analysis, Monte Carlo Simulation, Probability of failure*

1. INTRODUCTION

The mooring systems, transmission towers, offshore structures, and storage tanks require plate anchors to resist uplift forces. In the last few decades, the analysis of the uplift capacity of anchors in purely frictional soil got special attention. The uplift capacity of anchors in frictional soil was determined through a numerous investigations including both experimental and numerical methods.

Based on the results of laboratory tests, Meyerhof & Adams (1968) presented a general theory for the calculation of uplift capacity in frictional soil. From the small-scale laboratory tests, Das & Seeley (1975) proposed a proportional relationship between the critical embedment ratio with the length-width ratio of the anchor. Rowe & Davis (1982) investigated the behaviour of anchors in cohesionless soil. The effects of anchor embedment, friction angle, dilatancy, and roughness of the anchor were considered. In a medium to dense sand, the capacity of the anchor depended on the dilatancy of the frictional soil. According to Dickin (1988) and Murray & Geddes (1987), the increase in embedment ratio and soil density increased uplift resistance, and the length-width ratio reduced the uplift capacity of the anchor.

Several numerical analyses have been conducted over the last few decades to examine the uplift capacity of horizontal anchors. The interface roughness of the horizontal anchor plate was found to have a minor impact where the soil dilation impacted significantly on the uplift capacity (Merifield & Sloan, 2006). Bhattacharya & Kumar (2016) investigated the uplift capacity of anchors in layered sand using finite-element limit analyses. Khuntia & Prasad Sahoo (2018) analyzed the uplift capacity of strip anchors placed near the slopes in cohesive-frictional soil using finite-element lower-bound limit analyses. However, the effects of soil heterogeneity on the capacity and stability of horizontal strip anchors have not been examined to date. Even within a seemingly homogeneous frictional soil layer, soil strength characteristics vary significantly (Phoon & Kulhawy, 1999). The spatial variability of the soil strength parameter (i.e., soil friction angle) can be critical for the stability of the anchor and necessitates a detailed investigation. The purpose of this study is to investigate the effects of soil strength heterogeneity in spatially random frictional soil on the uplift capacity of horizontal strip anchor plate using finite element (FE) limit analyses and to suggest design factor of safety (FS) based on the heterogeneous characteristics of the soil.

In the current study, the uplift capacity of the horizontal strip anchor embedded in homogeneous purely frictional soil are examined for several embedment ratios (H/B, the ratio between embedment depth to plate width) using OPTUM G2 FE model. To convert the uplift capacity into a dimensionless parameter, break-out factors N_γ is used, defined as

$$N_\gamma = \frac{Q_o}{(B \times L)H\gamma} \tag{1}$$

Where: B = width of the strip anchor plate, L = length of the anchor plate (i.e., L=1 in strip anchor), H = depth of embedment, γ = unit weight of the frictional soil.

To validate the developed OPTUM G2 FE model, the obtained numerical outcomes are compared to the existing experimental and numerical results.

Further, the uplift capacity of the anchor is analysed using probabilistic numerical limit analyses to incorporate the stochastic spatial variation of friction angle in shallow depth (H/B = 1). The stochastic limit analysis is conducted based on the random field theory of the Karhunen-Loeve expansion with 1000 Monte Carlo Simulations. For the stochastic limit analyses in spatially random frictional soil, two variability parameters are considered (i.e., coefficient of variation of frictional angle, COV_φ, and isotropic correlation length of frictional angle, θ_φ).

The coefficient for the variation of friction angle (COV_φ) denotes the scattering data points in a computed data series around the mean friction angle, defined as

$$COV_\varphi = \frac{\sigma_\varphi}{\mu_\varphi} \tag{2}$$

Where: σ_φ, μ_φ are the standard deviation and the mean of friction angle, respectively.

The typical coefficient for the variation of friction angle (COV_φ) was found in the range of 5 to 11% (Phoon, 1995). The distance over which the spatial friction angle of cohesionless soil will be correlated is known as the spatial correlation length. Cami et al. (2020) found the range of vertical (θ_y) and horizontal (θ_x) correlation length in frictional soil. Table 1 shows the typical correlation length values in both vertical and horizontal directions for various types of frictional soil.

Table 1. The correlation length for various soil types (Cami et al., 2020)

Soil type	**Horizontal**	**Vertical**
	Average θ_x (m)	**Average θ_y (m)**
Marine Sand	15	1.43
Sand	24.5	1.17

In the current study, the isotropic correlation length ($\theta_x = \theta_y$) is considered with a constant mean friction angle (μ_{φ}= 30°). The isotropic correlation length of frictional angle (θ_{φ}) is converted dimensionless based on the study of Griffiths et al. (2002) and Kasama & Whittle (2011), defined as

$$\Theta_{\varphi} = \frac{\theta_{ln\varphi}}{B} \tag{3}$$

For analysing the effect of spatial variability of friction angle several combinations of COV_{φ}, Θ_{φ} are adopted in each analysis. For each set of assumed COV_{φ} and Θ_{φ}, Monte Carlo Simulation with 1000 repetitions has been conducted. The standard error of the anchor break-out factor for 1000 repetitions is about $1/\sqrt{n_{sim}} = 1/\sqrt{1000} = 0.03162 = 3.2\%$ of the standard deviation of each result, and the estimated variance have an error of $\sqrt{2/(n_{sim}-1)}$ = $\sqrt{2/(1000-1)}$ = 4.5% of the sample variance. Higher repetitions ensure minor error but significantly increases the computation time.

In shallow depth, Das & Seeley (1975) found the anchor break-out factor, $N_{\gamma\,(DAS)} = 1.7$, where H/B =1. The probability of the computed mean break-out factor N_{γ} is less than the value of 1.7 is found for various combinations of COV_{φ} and Θ_{φ}. For selected ranges of COV_{φ} and Θ_{φ}, the required design factor of safety (FS) is suggested based on probabilistic failure analyses.

2. PROBLEM DEFINITION

A horizontal strip anchor of a width, B = 2m, is placed into both homogeneous and heterogeneous frictional soil layer with a constant mean friction angle of 30°. In 2-D limit analyses, the length of the strip anchor is considered as, L=1m. A layout of the problem to be investigated is shown in Figure. 1.

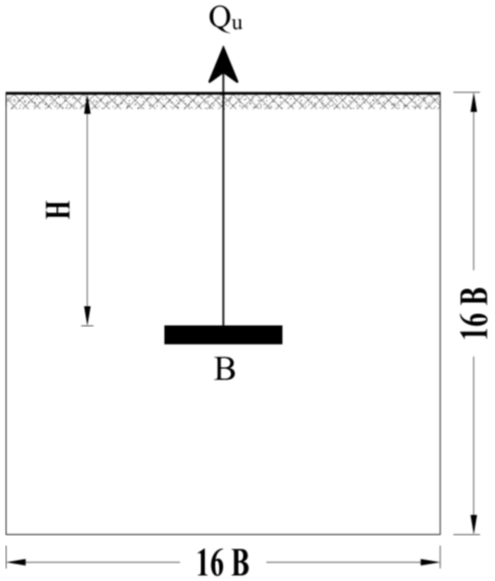


Figure 1. Problem definition of the horizontal strip anchor embedded in purely frictional soil

3. FINITE ELEMENT MODEL

The finite element (FE) limit analysis is carried out using OPTUM G2 FE model to determine the break-out factor of homogeneous frictional soil in each embedment ratio varying from 1 to 8 (H/B = 1, 2, 3, 4, 5, 6, 7, 8). The soil is assumed to be isotropic, and a Mohr-Coulomb yield criterion is adopted. The steel anchor plate is considered rough and weightless. Further, the failure probability of the strip anchor is analysed in spatially random frictional soil for shallow embedment ratio (i.e., H/B =1) for various combination of variability parameters.

A typical 2-D mesh of horizontal strip anchors in homogeneous frictional soil with a width of B, and an embedment ratio value of 7 (i.e., H/B = 7) is shown in Fig. 2. The boundary of the soil domain is extended 16B in both horizontal and vertical directions. The optimum mesh elements is chosen 6000 to ensure the accuracy of the results and reduce the computation time.

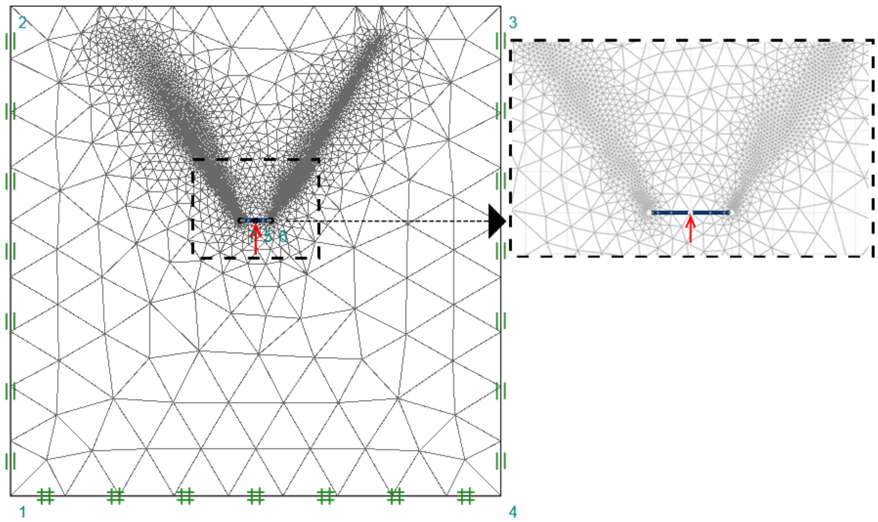
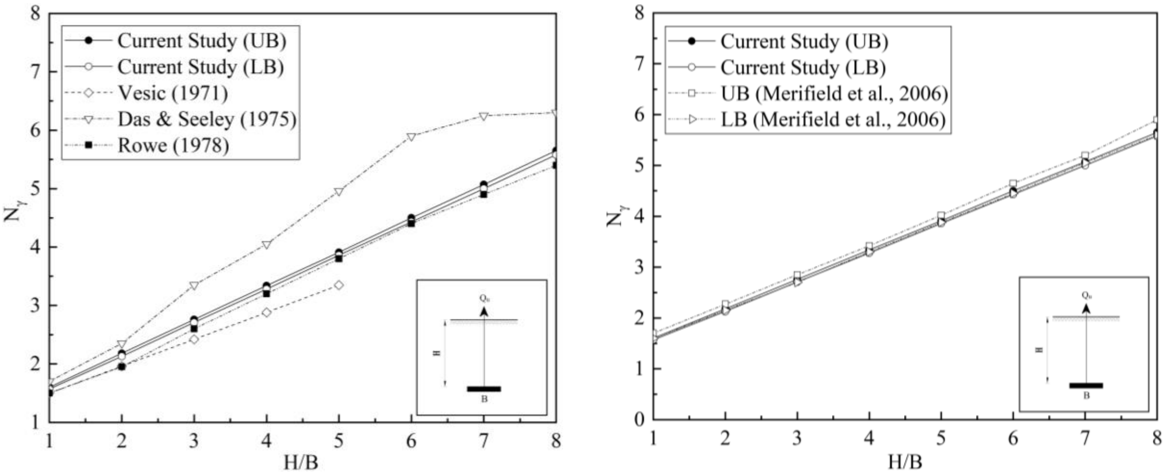


Figure 2. Lower Bound finite element mesh in homogeneous frictional soil (H/B = 7, B = 2m).

4. VALIDATION OF THE MODEL

Figure 3 compares the current numerical limit analysis with the available numerical and experimental results. In Fig. 3(a), the present results overestimate the outcomes of Rowe & Booker (1978) and Vesić (1971). The deviation of the results occurs because, Vesić (1971) considered the soil to be rigid-plastic near the surface and elastoplastic at greater depth, and Rowe (1978) investigated using a polished steel anchor with a length-width ratio, L/B =8.75. The present results deviate from the works of Das & Seeley (1975). The reason for the deviation is that, the uplift resistance of horizontal anchor was analysed with an aspect ratio, L/B = 5 and friction angle, φ = 31°, while the current study investigates the uplift capacity in strip anchor with mean friction angle value of 30°. According to Dickin (1988), the break-out factor diminishes as the aspect ratio (L/B) increases. From the laboratory test, Rowe (1978) found the uplift capacity of the rectangular anchor with L/B = 5 exceeds the strip anchor, where L/B = ∞. The obtained break-out factor in the current study agrees with the work of Dickin (1988) and Rowe & Booker (1978). For a strip anchor, the break-out factor is lower than the rectangular anchor in the same embedment ratio. However, the current results approximate pretty well with the numerical study of Merifield & Sloan (2006), as evident in Fig. 3(b). The exact value of break-out factors is somewhere within ±1% of the upper bound (UB) and lower bound (LB) solutions.



(a) (b)

Figure 3. Break-out factors for horizontal strip anchor in frictional soil: (a) comparison with the available laboratory tests; (b) comparison with existing numerical results.

5. NUMERICAL LIMIT ANALYSIS WITH RANDOM FIELD DISTRIBUTION

The upper and lower bound limit analysis is performed using a constant mean friction angle, μ_{φ} = 30°, while the combination of COV_{φ}, Θφ are varied using the following ranges:

COV_{φ} = 0.05, 0.1, 0.15, 0.25, 0.5, 0.75, 1.0

Θ_{φ} = 0.0625, 0.125, 0.25, 0.5, 1, 2, 4, 8, 16

1000 Monte Carlo Simulations are analysed for each combination of coefficient of variation and correlation length. The mean and standard deviation of break-out factor is calculated by,

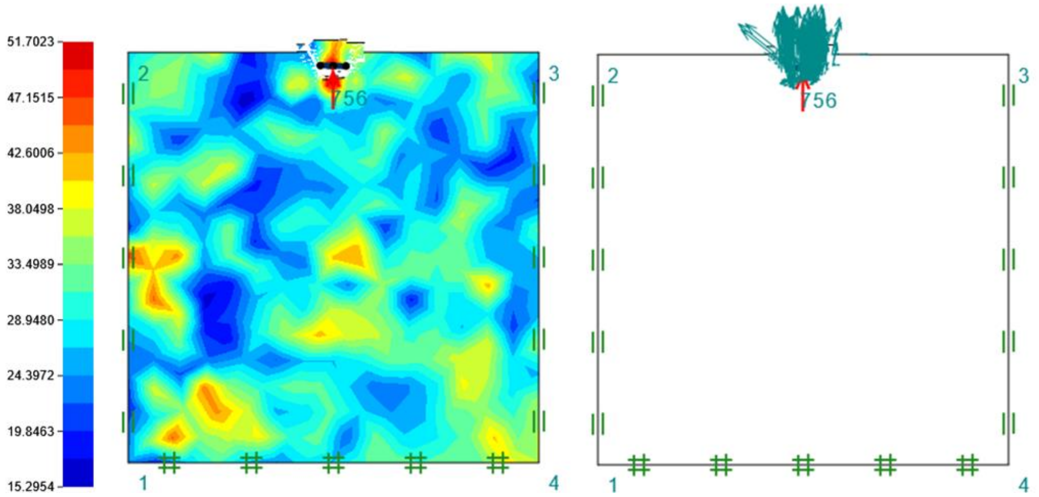
$$\mu_{N_\gamma} = \frac{1}{1000 \times H \times \gamma} \sum_{1}^{1000} \left(\frac{Q_{oi}}{B}\right) \qquad i = 1, 2, 3, 4, \ldots\ldots\ldots\ldots, 1000 \tag{4}$$

$$\sigma_{N_\gamma} = \sqrt{\frac{1}{1000-1} \sum_{i=1}^{1000} (N_{\gamma i} - \mu_{N_\gamma})^2} \qquad i = 1, 2, 3, 4, \ldots\ldots\ldots\ldots, 1000 \tag{5}$$

The break-out factor ratio (Nγ′) is computed by,

$$N_{\gamma}' = \mu_{N\gamma} / N_{\gamma\,(DAS)};\ N_{\gamma(DAS)} = 1.7 \tag{6}$$

Figure 4 shows a typical deformed shape of horizontal plate anchor at failure corresponding to heterogeneous frictional soil, where COV_{φ} = 0.25, Θ_{φ} = 1, H/B=1 and μ_{φ}= 30°, the isotropic correlation length is 2m (θ_x = θ_y = 2m). The reddish zone denotes dense soil, whereas the bluish region indicates loose soil. Figure 4(b) shows a non-symmetric failure pattern of the plate anchor.



(a) (b)

Figure 4: Typical lower bound limit analysis in heterogeneous frictional soil: (a) deformed shape of anchor at failure with $COV_{\varphi} = 0.25$, $\Theta_{\varphi} = 1$; (b) displacement vectors.

In case of $COV_{\varphi} = 0.25$, $\Theta_{\varphi} = 1$, a mean break-out factor, $\mu_{N_\gamma} = 1.4832$ is calculated using equation (4). Due to the spatial variation of friction angle, a reduced break-out factor of 5.85% is found from the obtained Nγ in the homogeneous soil where, H/B = 1, Nγ = 1.57.

5.1 Study of Log-Normal Distribution

Random log-normal variability parameters (i.e., COV_{φ}, Θ_{φ}) is used to incorporate spatial variation of friction angle, where ln_{φ} is normally distributed. The log-normal distribution of σ_{φ}, μ_{φ} are given by

$$\sigma_{ln_\varphi} = \sqrt{\{ln[1 + (COV_\varphi)^2]\}} \tag{6}$$

$$\mu_{ln_\varphi} = ln\mu_\varphi - \frac{1}{2}\sigma_{ln_\varphi}^2 \tag{7}$$

The probability distribution function (PDF) is given by

$$f\ (x;\ \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} exp[\frac{-(ln\,x-\mu)^2}{2\sigma^2}], x > 0 \tag{8}$$

To incorporate spatial variability parameters, the current study uses log-normal function because it is always non-negative.

6. RESULTS OF FINITE ELEMENT LIMIT ANALYSIS

For each set of input values (i.e., COV_{φ} and Θ_{φ}), the mean and standard deviation of the obtained break-out factors are determined using equations (4) and (5). A 22 bins histogram of 1000 break-out factors is plotted with the 'best-fit' log-normal distribution, shown in figure 5; where $COV_{\varphi} = 0.25$, $\Theta_{\varphi} = 1$.

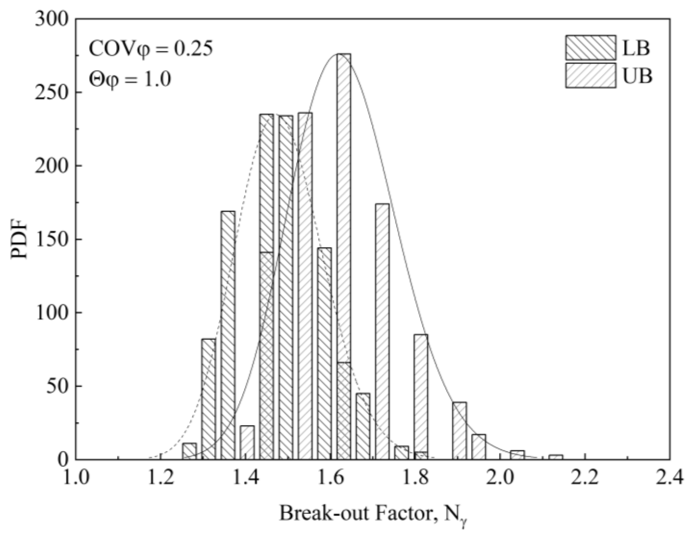
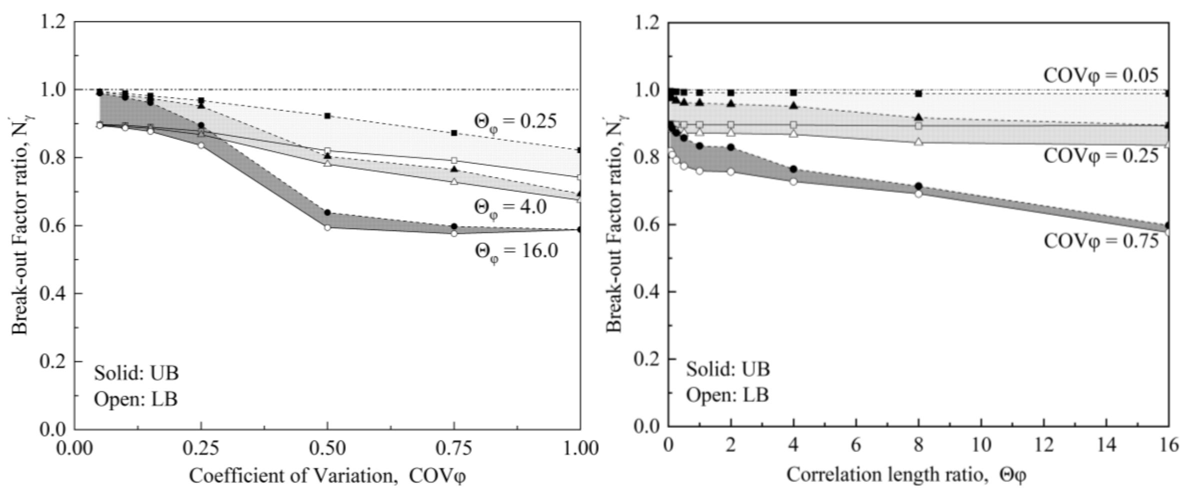


Figure 5: Histogram and log-normal fit of the break-out factors (COV_φ = 0.25, Θ_φ = 1).

Figure 6 indicates the effect of COV_φ and Θ_φ on the break-out factor ratio (Nγ′) for an embedment ratio of H/B=1. For an increase in COV_φ, the estimated ratio of Nγ′ decreases. The decrease rate is higher for $COV_\varphi \leq 0.5$ and larger Θ_φ but smaller for $COV_\varphi > 0.75$ and larger Θ_φ, shown in Figure 6(a). For $COV_\varphi > 0.5$ and $\Theta_\varphi = 16$, the reduction rate of Nγ′ is minimal. The impact of dimensionless correlation length ratio of friction angle on the break-out factor ratio is shown in Figure 6(b). For higher values of Θφ, the ratio of Nγ′ falls steeply. The reduction rate also increases with an increase of coefficient of variation of friction angle. A very small Nγ′ is found for the combination of COV_φ = 0.75, Θ_φ = 16.



(a) (b)

Figure 6: (a) Mean break-out factor ratio Nγ′ as a function of COV_φ and Θ_φ. (b) variation of correlation length ratio with Nγ′.

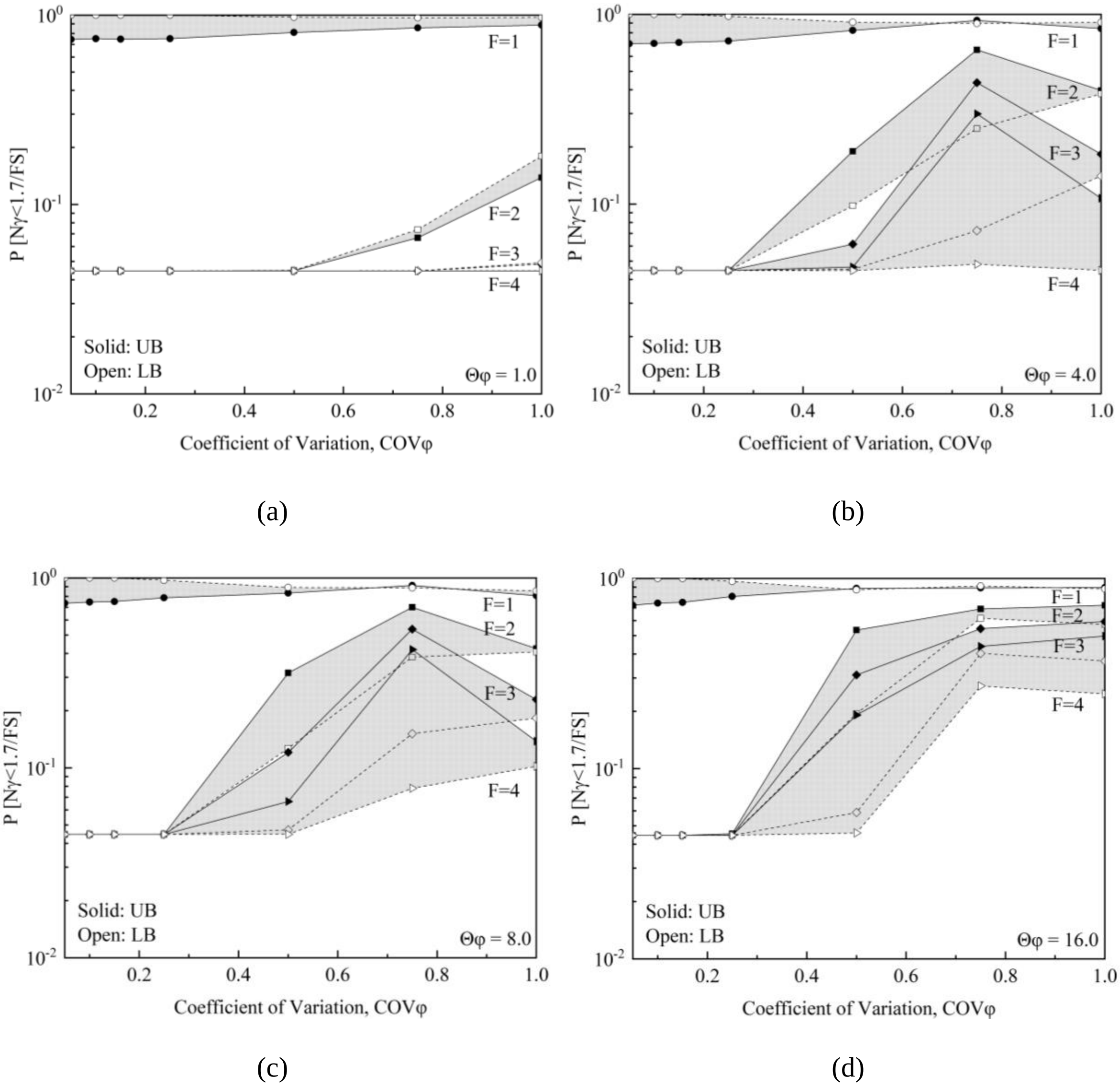


Figure 7: Probability that the mean break-out factor is less than the nominal design factor: (a) Θ_φ =1; (b) Θ_φ =4; (c) Θ_φ =8; (d) Θ_φ =16.

6.1 Failure probability of plate anchor

Design failure assume to occur when the estimated mean break-out factor is less than the suggested nominal break-out factor, $N\gamma_{\text{(DAS)}}$ = 1.7.

For lognormally distributed Nγ, the probability of this failure P [Nγ < 1.7/FS] is given by,

$$P\left[N\gamma < \frac{1.7}{FS}\right] = \Phi\left(\frac{ln(1.7/FS) - \mu_{ln_{N\gamma}}}{\sigma_{ln_{N\gamma}}}\right) \tag{9}$$

Where $\Phi(\ldots) =$ cumulative normal function. For a particular set shown in Figure 5, $\mu_{N\gamma} = 1.6336\ (UB)$ and $\sigma_{N\gamma} = 0.1297\ (UB)$. Equations (6) and (7) give $\sigma_{ln_{N\gamma}}$= 0.07927051 and $\mu_{ln_{N\gamma}} = 0.48764426$. From equation (9), the probability of failure is found as P [Fc < 1.7/1] = 0.7508, indicating a 75.08 % probability that the computed uplift capacity will be less than the design capacity. The lower bound solution indicates a 99.5 % probability that the uplift capacity is less than the nominal factor. The exact probability of failure will be in between 75 to 99 %.

Figure 7 summarizes the effect of factor of safety (i.e., FS = 1, 2, 3, 4) with the probability of design failure as a function of coefficient of variation for several dimensionless correlation length ratio, Θ_φ = 1, Θ_φ = 4, Θ_φ = 8, Θ_φ = 16. The results reveal that higher FS is necessary to reduce the probability of failure. For purely frictional soil with COV_φ = 0.25 and $\Theta_\varphi \leq 16$, a safety factor of at least two is required, while greater FS is required for large COV_φ. Fig. 7 also shows, more significant FS is necessary with an increase of Θ_φ. For COV_φ = 0.5 and $\Theta_\varphi \geq 8$, the design factor of safety value of 4 is required.

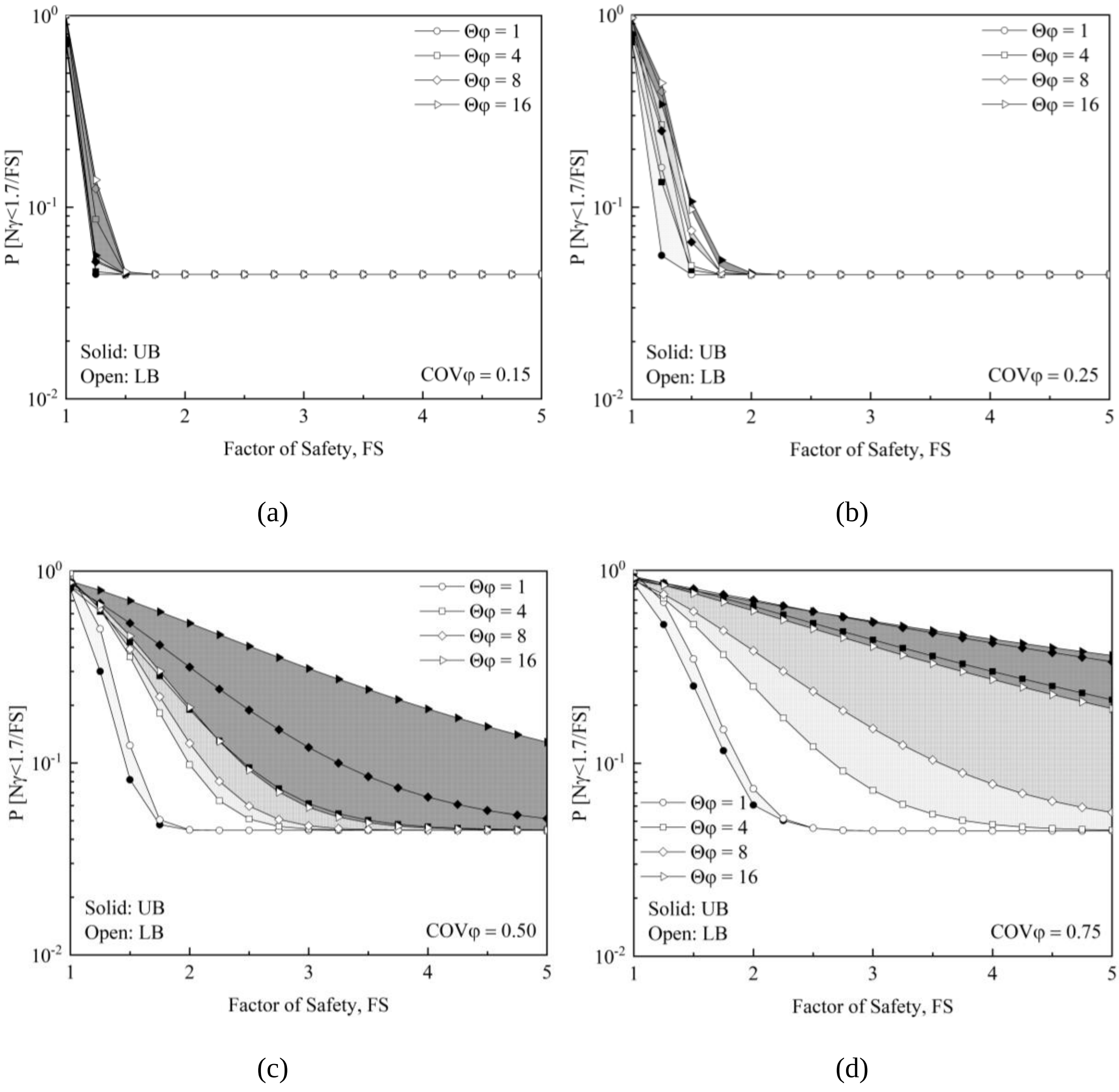


Figure 8: Variation of P [Nγ < 1.7/FS] with safety factor for a frictional soil: (a) COV_φ = 0.15, $1 \leq \Theta_\varphi \leq 16$; (b) COV_φ = 0.25, $1 \leq \Theta_\varphi \leq 16$; (c) COV_φ = 0.5, $1 \leq \Theta_\varphi \leq 16$; COV_φ = 0.75, $1 \leq \Theta_\varphi \leq 16$.

Figure 8 depicts a more detailed probabilistic interpretation of design failure with the variation of safety factors. For an embedment ratio of H/B =1, a failure probability value of 4.4% always exists. This is because, shallow anchors in loose sand will fail along a sloped surface. This sloped surface propagates almost linearly in a shallow embedment depth. Figure 8 shows that P [Nγ < 1.7/FS] is acceptable for $COV_\varphi \leq 0.25$, $\Theta_\varphi \leq 16$ with a safety factor value of 2. When $0.25 < COV_\varphi \leq 0.5$, $1 \leq \Theta_\varphi \leq 4$, a minimum FOS value of 4 is required. For $0.25 < COV_\varphi \leq 0.5$, $\Theta_\varphi > 4$, FS value of 5 is necessary. However, in rare cases with $0.5 \leq COV_\varphi \leq 0.75$ and $1 < \Theta_\varphi \leq 16$, a higher FS is necessary.

7. CONCLUSIONS

This study investigates the effects of spatial variability of friction angle on the uplift capacity of the shallow horizontal strip anchor embedded in spatially random frictional soil. The spatial variation of friction angle is assumed to be represented by the coefficient of variation (COV_{φ}) and dimensionless isotropic correlation length ratio (Θ_{φ}). The use of lower and upper bound limit analysis ensures the exact solution in between UB and LB results., The following conclusions can be drawn based on the outcomes of this study:

(a) The calculated mean break-out factor drops as the coefficient of variation (COVφ) increases. For higher value of Θ_{φ}, the mean break-out factor falls steeply.
(b) The impact of correlation length on the probabilistic uplift capacity interpretations is significant, particularly for a heterogeneous frictional soil with a greater coefficient of variation.
(c) As the coefficient of variation increases, the necessity of higher safety factor increases. A more significant factor of safety (FS) is required when $COV_{\varphi} > 0.25$.
(d) By investigating the effect of the probability of failure with the safety factor, a FS value of two is suggested when $COV_{\varphi} < 0.25$, $\Theta_{\varphi} < 16$. A minimum safety factor value of four is necessary for a cohesive soil with $0.25 < COV\varphi < 0.5$, $1 \leq \Theta\varphi \leq 16$. However, in exceptional cases where $COV_{\varphi} \geq 0.5$ and $4 \leq \Theta_{\varphi} \leq 16$, a higher FS is required.
(e) More investigation is needed to establish the impact of anisotropic correlation length and the coefficient of variation of friction angle on the uplift capacity of deep horizontal anchors embedded in frictional soil.

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